Global Geodynamics:
Free Oscillation,
Earth Rotation,
Gravity Field

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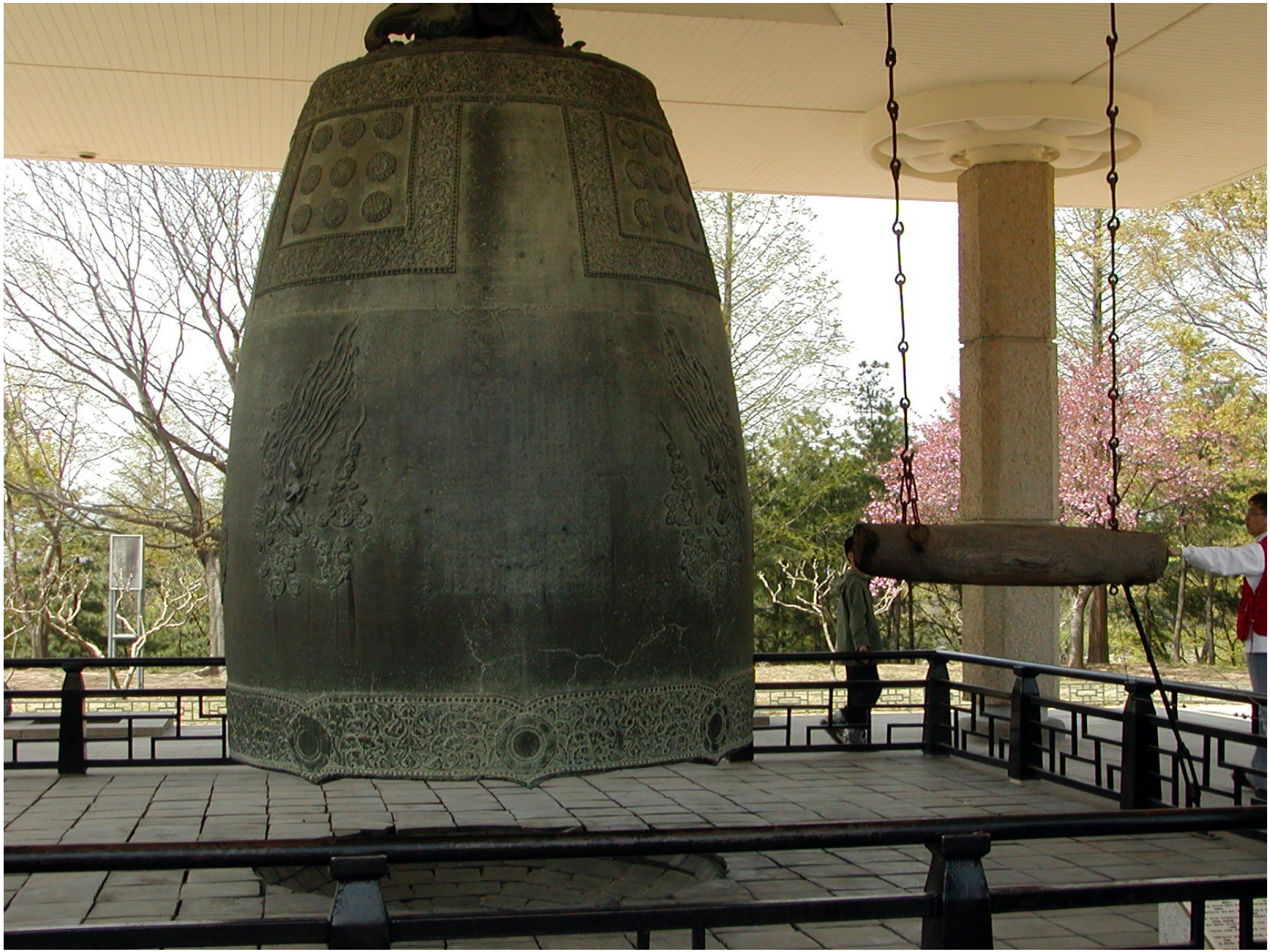


1-D traveling-wave – standing-wave

•equivalence

$$\sin(wt + x) + \sin(wt - x)$$

$$= 2 \sin wt * \cos x$$

A typical seismogram

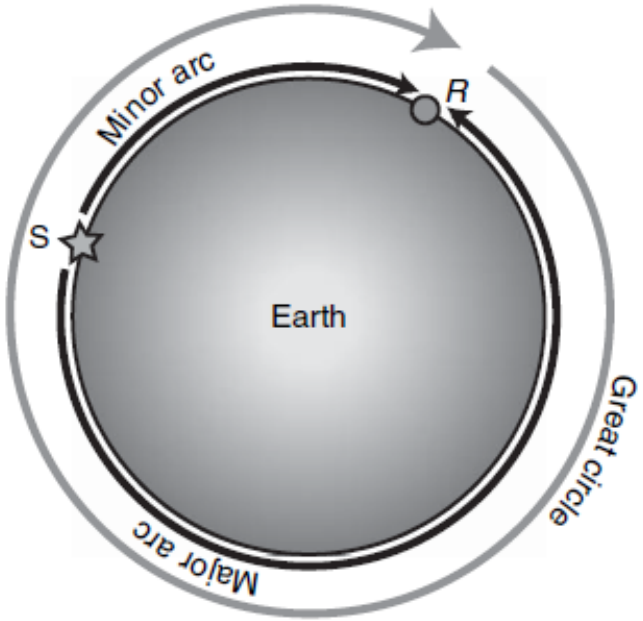
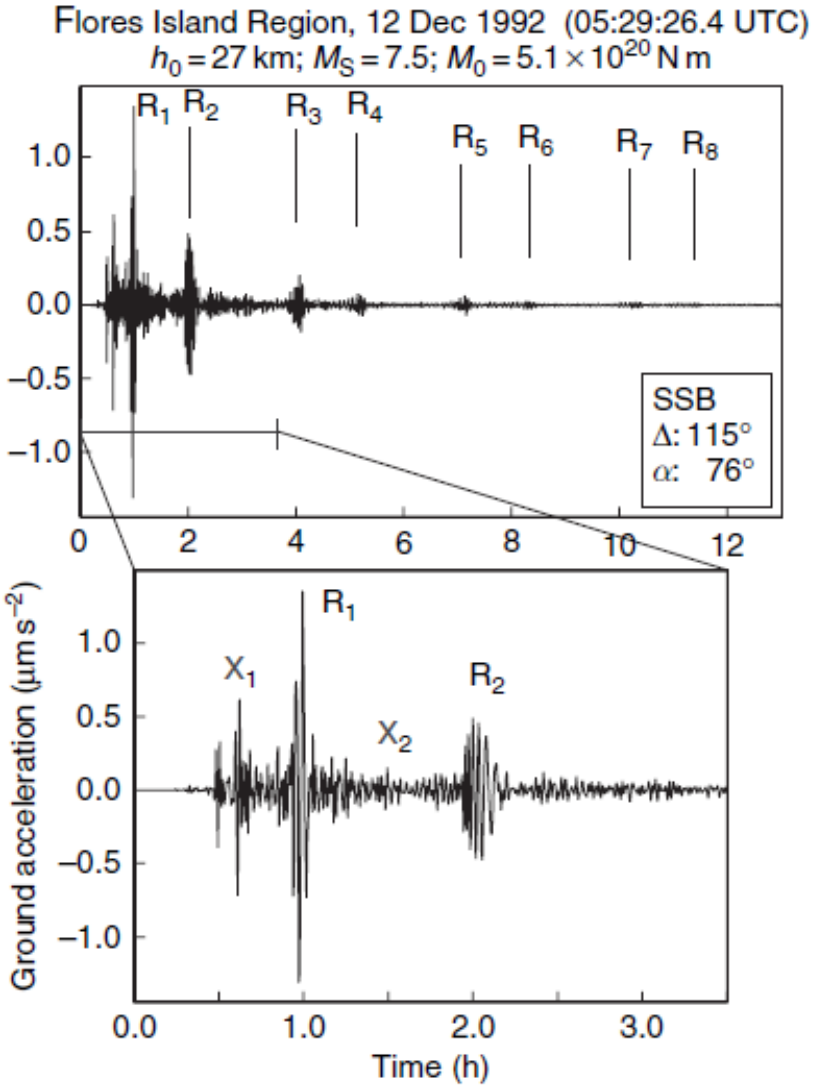


Figure 1 Surface wave paths from source S to receiver R. Paths with distance less than 180° are minor arc paths (R_1), paths with larger distances are major arc paths (R_2). The number following R is the wave orbit number. The wave orbit number advances by two when successive wave trains include a complete great circle (e.g., R_3 and R_5 for the minor arc).

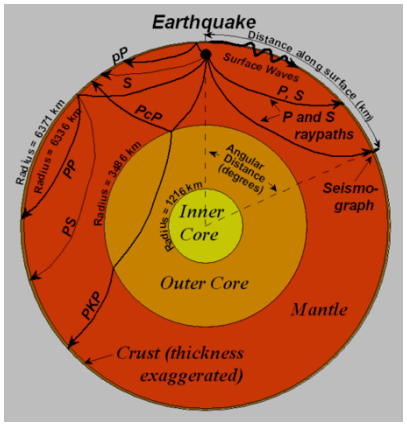


Free Oscillations -- Physically

- free oscillation = "music".
- A 3-D finite elastic body, the Earth is a "resonating" musical instrument with characteristic **modes** of free oscillation, or resonance standing waves (can be treated equivalently as propagating waves, such as in ray theory).
- The "tone" of the music

  = the "harmonic" eigen-frequencies

  with Q-values (for decay).



- The amplitude and phase of the music are "excited" by a source, most often large earthquakes.

Free Oscillations -- Mathematically

- Eigen-solutions of the wave equation with proper boundary conditions,
  - "harmonic" (sinusoidal) in time
- Modes are:
  - Standing waves in the form $\boldsymbol{S}_i(\boldsymbol{r})T_i(t)$
  - Orthogonal, or "**normal**" (inner products are Kronecker)
  - Complete (expand any space-time function)

Free Oscillations of SNREI Earth

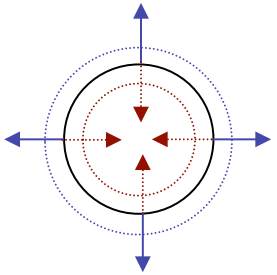
- Space (3-D) : n (overtone), l (degree), m (order):
- l,m => spherical harmonics
- n => radial eigenfunction:
  - Spheroidal U_n , V_n
  - Toroidal W_n
  - $n = 0$: fundamental modes
  - $n \neq 0$: overtone modes
- ${}_n\omega_{lm}$ are $2l+1$-fold degenerate w.r.t. m (-l,…0…,l), because of spherical symmetry
- Time (1-D): eigrnfrequencies ${}_n\omega_{lm,}$ discrete because of boundary conditions

Spheroidal modes

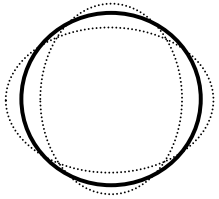
•corresponding to P/SV and Rayleigh wave motion

•horizontal,+ vertical (hence gravitational and volume effect)

•$_0S_0$: "Breathing" mode, periods ~20 minutes



•$_0S_2$: "Rugby" mode, periods ~54 minutes

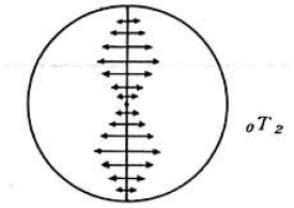


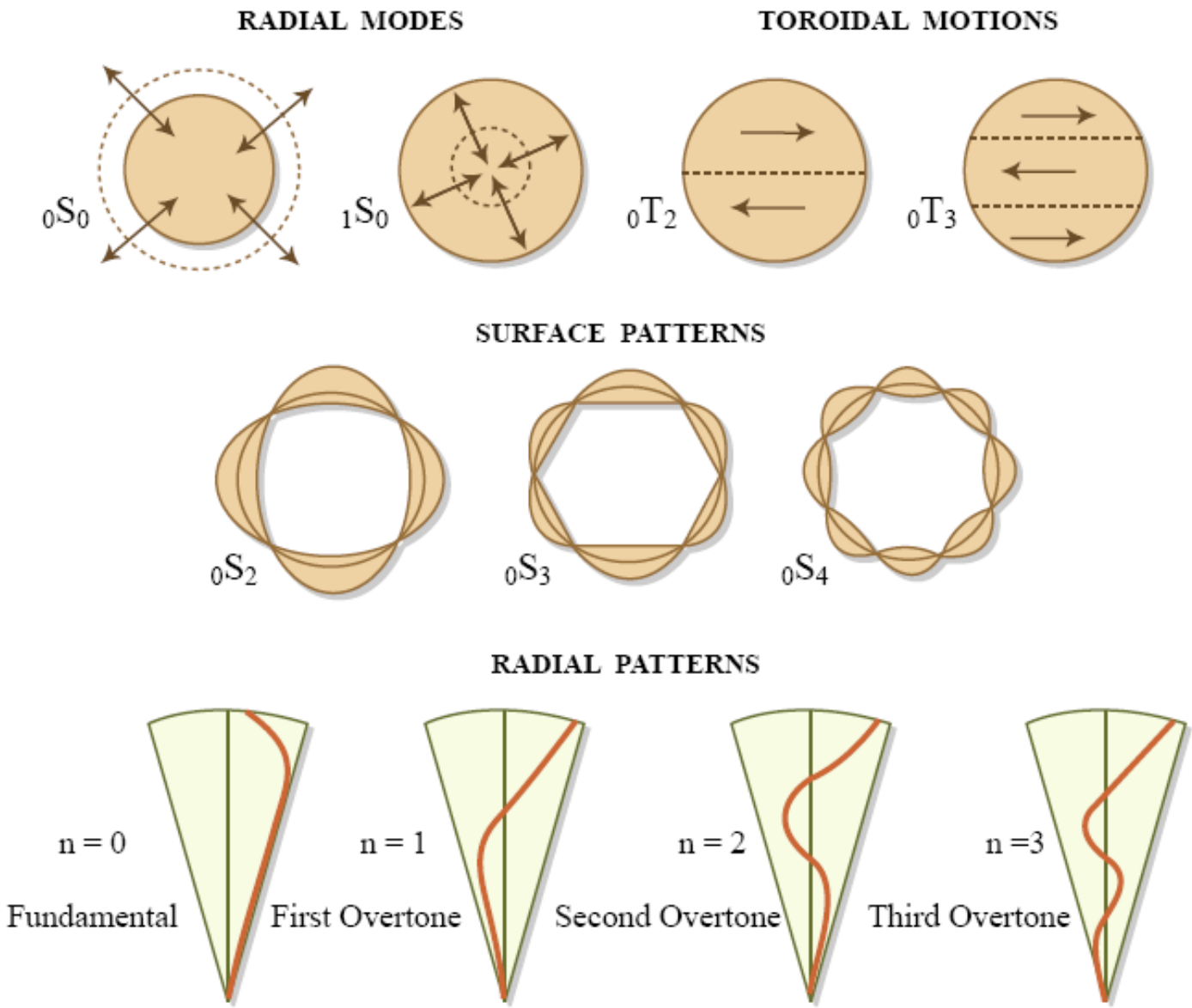
• $_0S_1$: shift in the center of mass of the Earth, cannot be excited by earthquakes (an internal force)

•$_1S_1$: Slichter triplet, translation of inner core, periods ~4-6 hours?

Toroidal modes

- cooresponding to SH Love wave motion
- horizontal only
- ${}_0T_0$: has no physical meaning and is undefined
- ${}_0T_1$: change in Earth's rotation, , cannot be excited by earthquakes (an internal force)
- ${}_0T_2$: twisting between the northern and southern hemispheres, periods ~44 minutes




RADIAL MODES
TOROIDAL MOTIONS
$_0S_0$
$_1S_0$
$_0T_2$
$_0T_3$
SURFACE PATTERNS
$_0S_2$
$_0S_3$
$_0S_4$
RADIAL PATTERNS
n = 0
Fundamental
n = 1
First Overtone
n = 2
Second Overtone
n =3
Third Overtone

Inversion for a 1-D Earth model

- All eigen-periods (and *Q*-values) are functionals of Earth's density, rigidity and shear modulus (and viscosity).
- Non-unique.
- Used as 1st-order (linear) "perturbation" to existing 1-D models (Taylor expansion), to correct/improve the latter

  => 1066A, 1066B, PREM,…

Earthquake Displacement Field

•• Equation of motion

$$\nabla\tau + \mathbf{f_g} + \mathbf{f_s} = \rho(\mathbf{r})\frac{\partial^2 \mathbf{u}}{\partial t^2}$$

•• Solve by expanding displacement field

$$\mathbf{u}(\mathbf{r},t) = \sum_k a_k(t)\,\mathbf{u}_k^*(\mathbf{r})$$

- Normal mode eigenfunctions for SNREI Earth

$$
\begin{aligned}
{}_n u_\ell^m(\mathbf{r},\,t) = \mathrm{Re}\big[\big(\hat{\mathbf{r}}\,{}_n U_\ell(r)\,Y_\ell^m(\theta,\,\phi) + {}_n V_\ell(r)\nabla_1 Y_\ell^m(\theta,\,\phi) \\
- {}_n W_\ell(r)\hat{\mathbf{r}}\times\nabla_1 Y_\ell^m(\theta,\,\phi)\big)\,e^{i\,{}_n\omega_\ell^m t}\big]
\end{aligned}
$$

- Expansion coefficients (note the static limit)

$$a_k(t) = \frac{M_O}{\omega_k^2}\,\hat{\mathbf{M}}{:}\mathrm{E}_k^*(\mathbf{r_s})[\exp(i\omega_k t) + 1]$$

(Gilbert, 1970)

Seismic Moment Tensor

- Contains all information about source mechanism (magnitude, fault direction, slip angles, etc.)
- 2nd-order tensor (conservation of linear momentum)
- Symmetric tensor, only 6 independent parameters (conservation of angular momentum)
- Magnitude (seismic moment) sqrt(2)*[M:M]^{½} is a good measure of earthquake size

  => moment scale (vs. Richter scale)

Co-Seismic Displacement Field

$$\mathbf{u}(\mathbf{r}) = \text{oscillations} + \text{static displacement}$$

$$= 0 \ (\text{as } t \rightarrow \infty) + \Sigma_{k=0}^{\infty} \omega_k^{-2} \, \mathbf{u}_k(\mathbf{r}) \, \mathbf{M} : \mathbf{E}_k^*(\mathbf{r}_f)$$

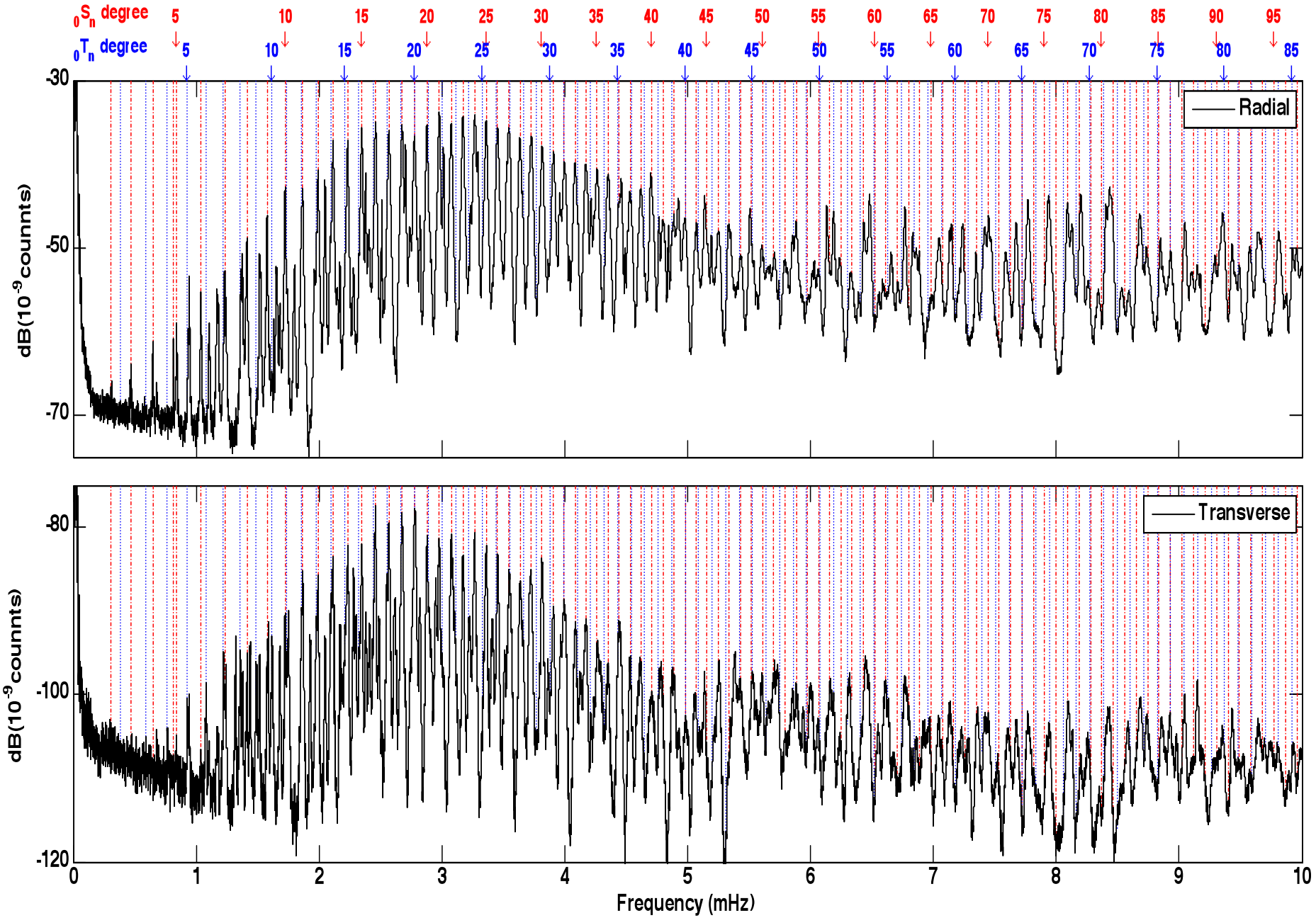
Eigen-mode ($\mathbf{u}_k(\mathbf{r})$, $\mathbf{E}_k(\mathbf{r}_f)$, ω_k , k = spheroidal and toroidal:

from SNREI model (e.g., 1066A, B; PREM)

Moment tensor **M**: from Global CMT catalog

Free Oscillations of (perturbed) non-SNREI Earth

- Elasto-gravitational, periods < 1 hour
- Modes no longer "normal" (generalized orthogonality) due to rotation (involving centrifugal and Coriolis forces)
- ${}_n\omega_{lm}$ shifted (but diagonal sum rule)
- ${}_n\omega_{lm}$ split (break of symmetry) by:
  - Rotation – Zeeman type
  - Ellipticity – Stark type
  - Lateral heterogeneity
- Mode-coupling by the above (selection rules)


$_0S_n$ degree
$_0T_n$ degree
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
-30
-50
-70
dB(10^{-9}counnts)
Radial
-80
-100
-120
Transverse
0
1
2
3
4
5
6
7
8
9
10
Frequency (mHz)

${}_1S_1^{-1,0,1}$ "Slichter triplet"

= The 3 translational oscillations of the solid inner core inside the liquid outer core (Slitcher, 1962)

The 3 split triplet frequencies should be on the order of several hours according to seismological models of the Earth.

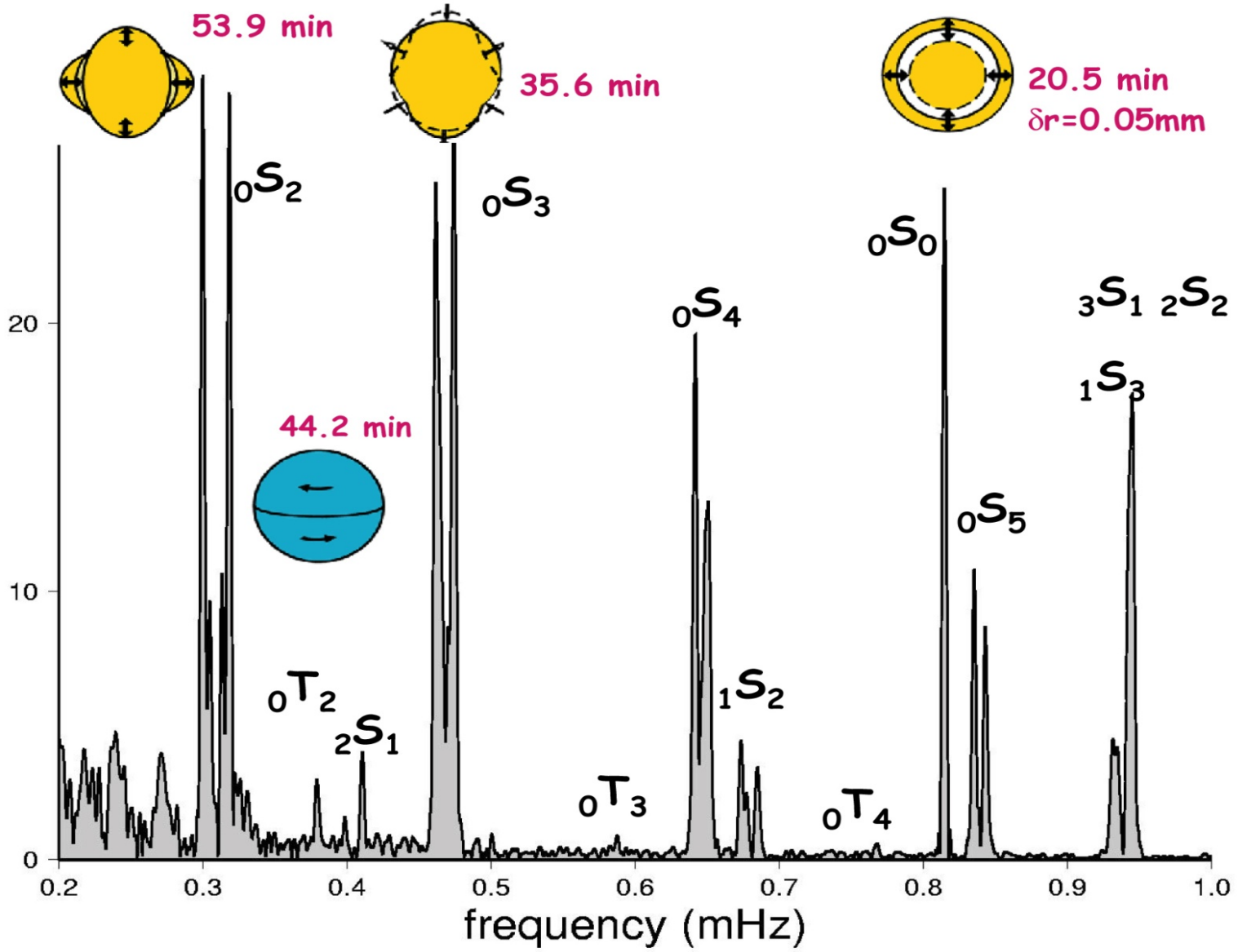
The (complex) frequencies are functionals of the inner core - outer core density contrast at the inner core boundary (ICB), the viscosity at the ICB, and the stratification of the outer core.

- => Useful for inversion of core structure and dynamics

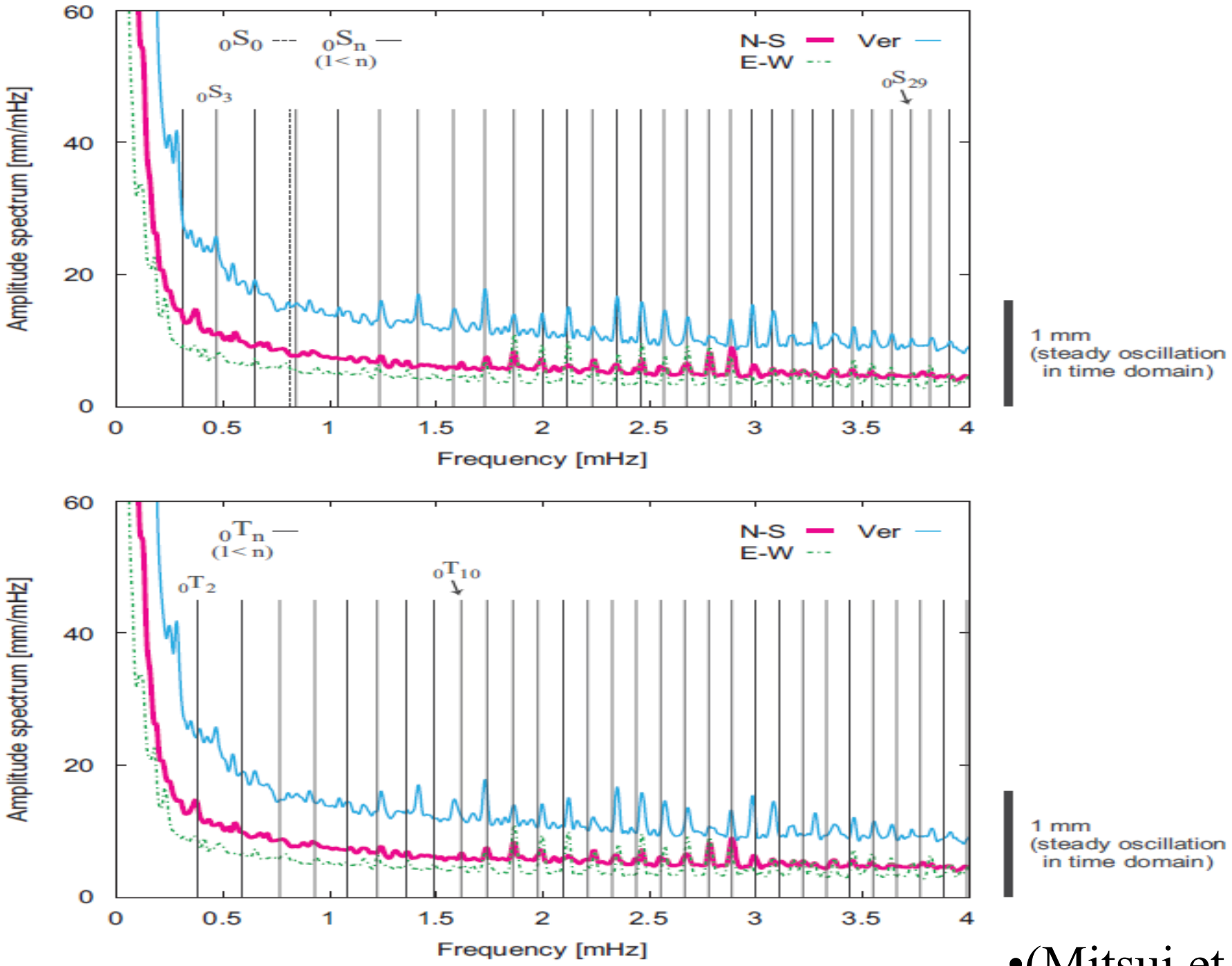
"The Earth precesses/nutates like a top."

"The Earth librates like a physical pendulum."

"The Earth wobbles like a frisbee."



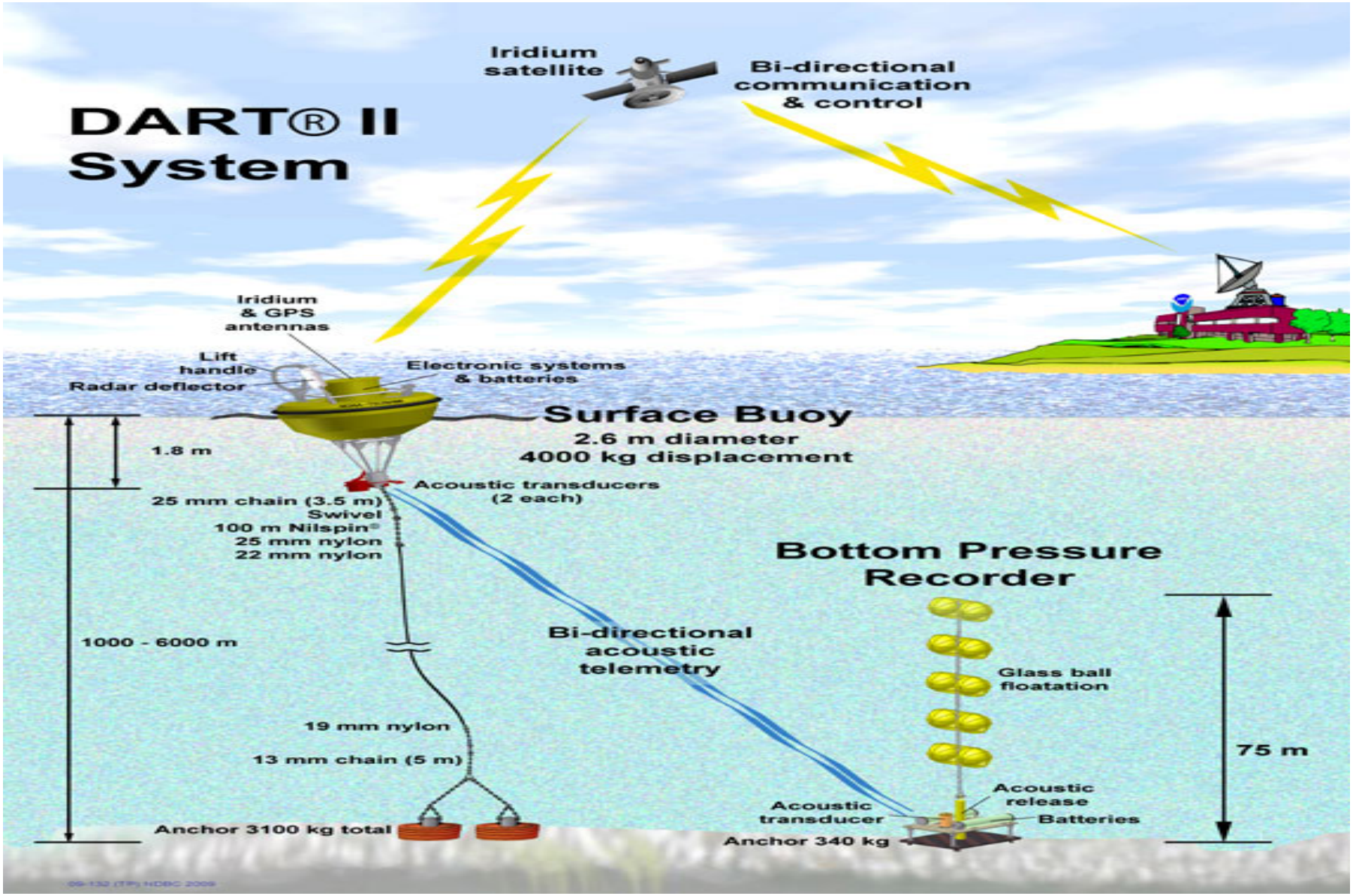
•(Park et al., 2005)

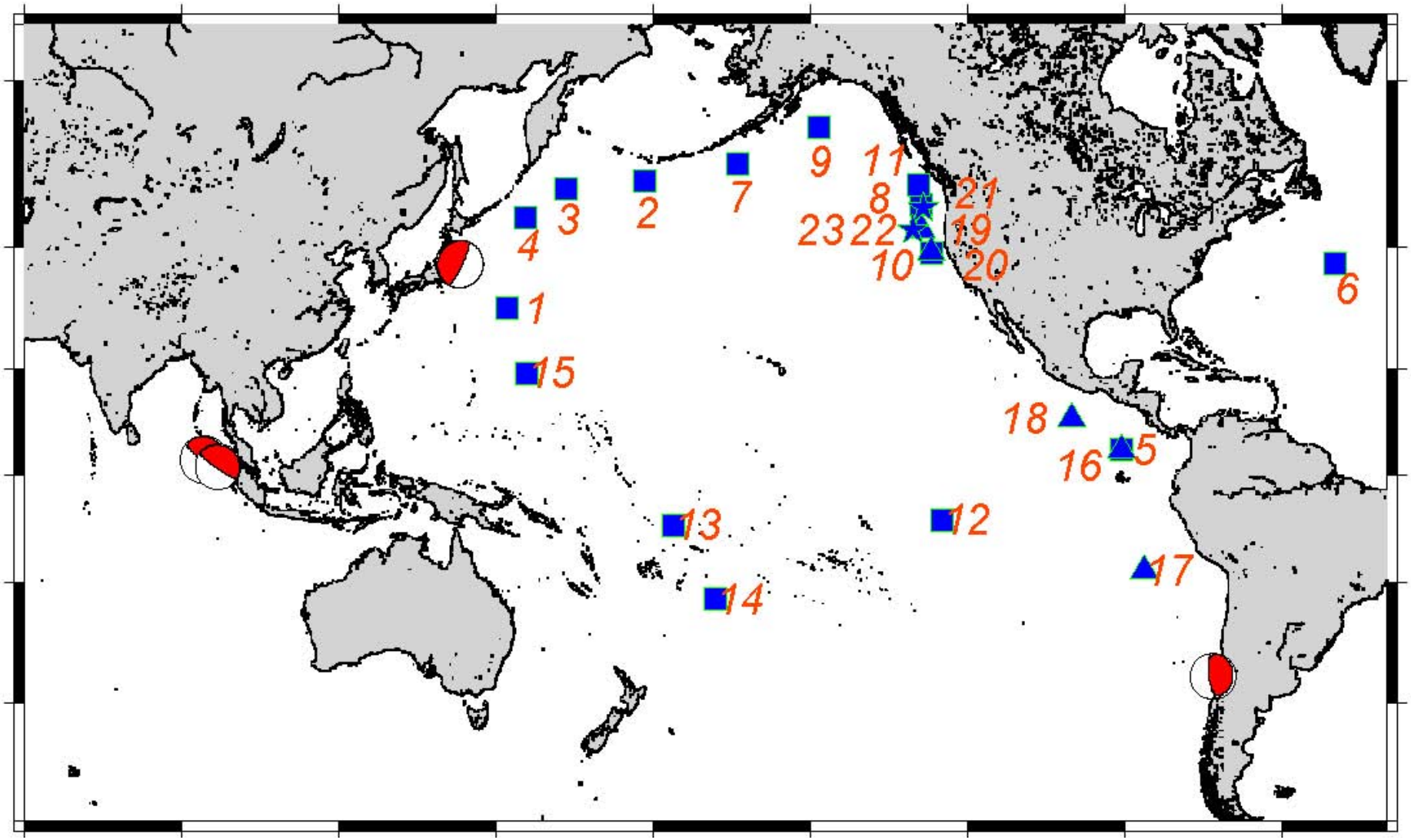


•(Mitsui et al., 2

DART of NOAA

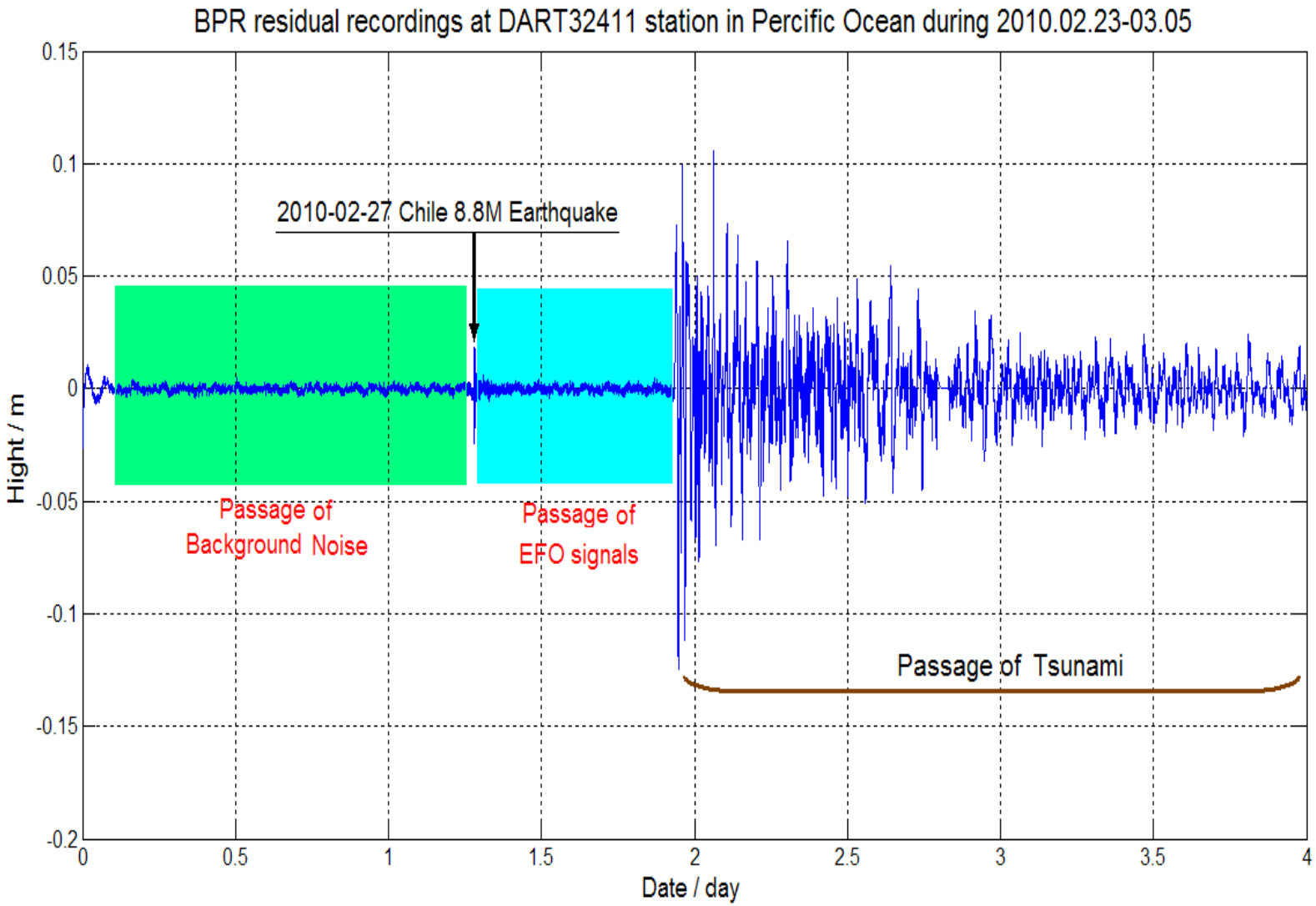
Deep-ocean Assessment and Reporting of Tsunamis network



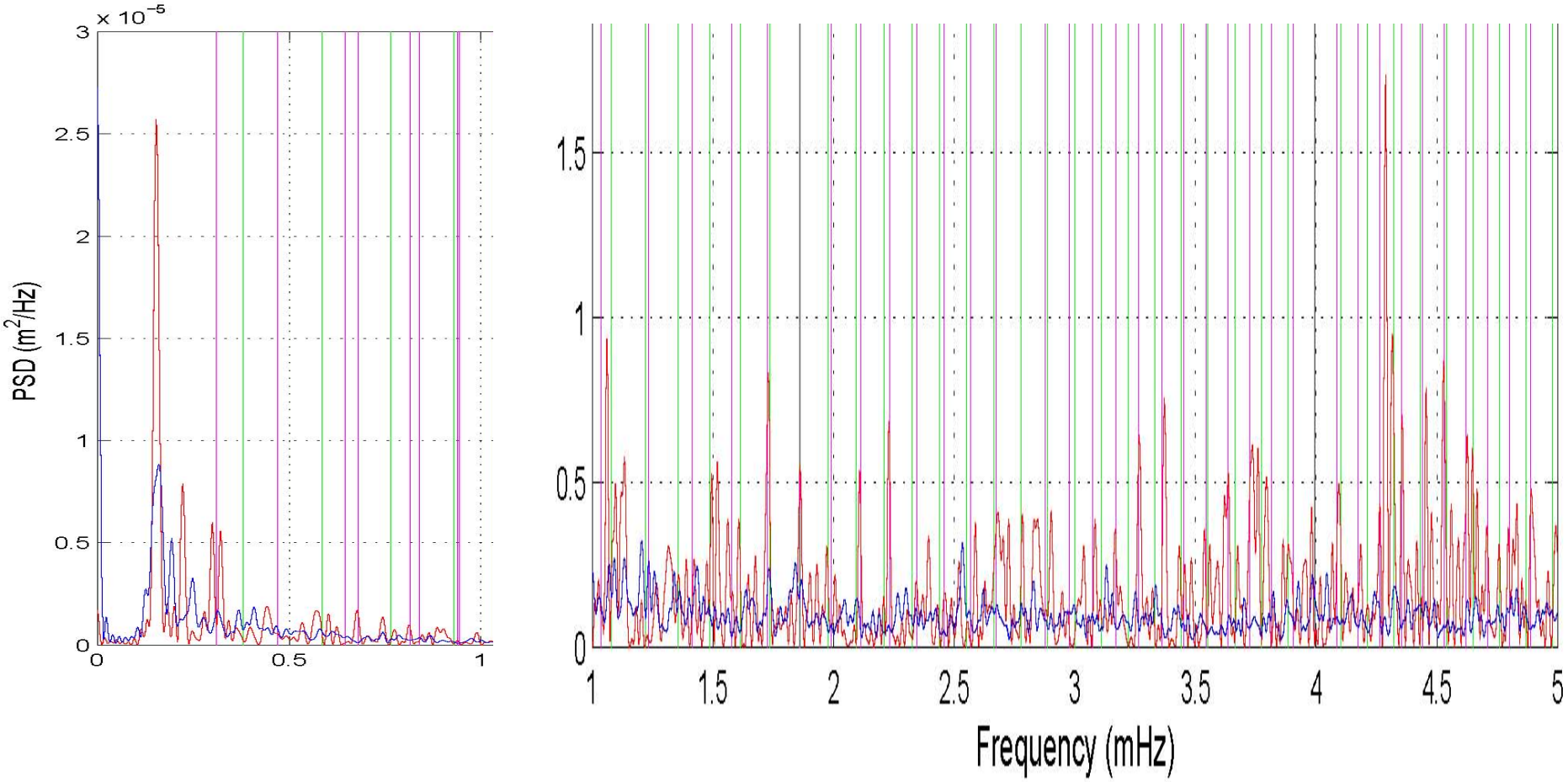


•24 BPR records used (15 for 2010 Chile, 5 for 2011 Japan, 3 for 2005 Sumatra/ Nias, 1 for 2004 Sumatra): After arrival of seismic waves but before tsunami, typically several hours long, at “event mode” 15-sec sampling.

A typical BPR earthquake record (4 days long, after de-tide)



Spectrum of a single DART (BPR21413) record for 2010 Chile earthquake



- **Blue: pre-earthquake**
- **Pink line: spheroidal $_0S_n$ modes**

Red: during earthquake

Green line: toroidal $_0T_n$ modes

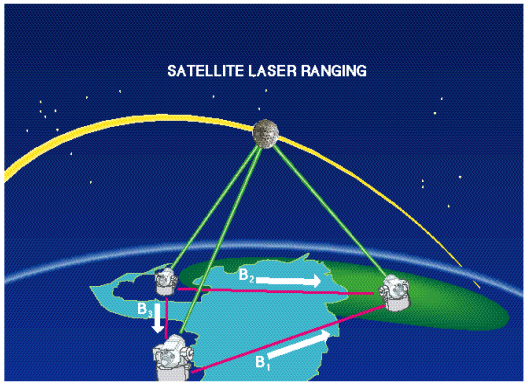
Rigid-body modes

- 6 trivial solutions to the normal mode equation
- For non-rotating Earth, “zero-frequency”:
  - 3 for translation ($_0S_1$)
  - 3 for rotation ($_0T_1$)
- For rotating Earth:
  - 3 for translation
  - 3 for rotation (with non-zero frequency), -- axial spin, tilt-over, Euler-Chandler wobble, no longer “normal”, but generalized orthogonal under Coriolis term

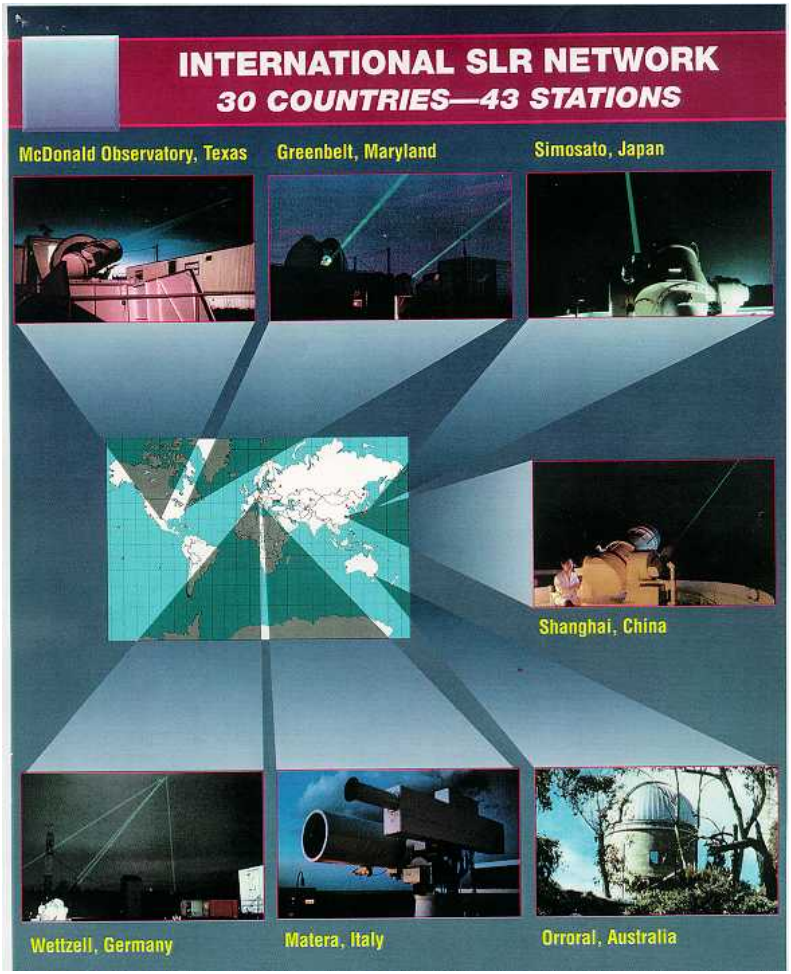
Earth's Rotation, 3-D

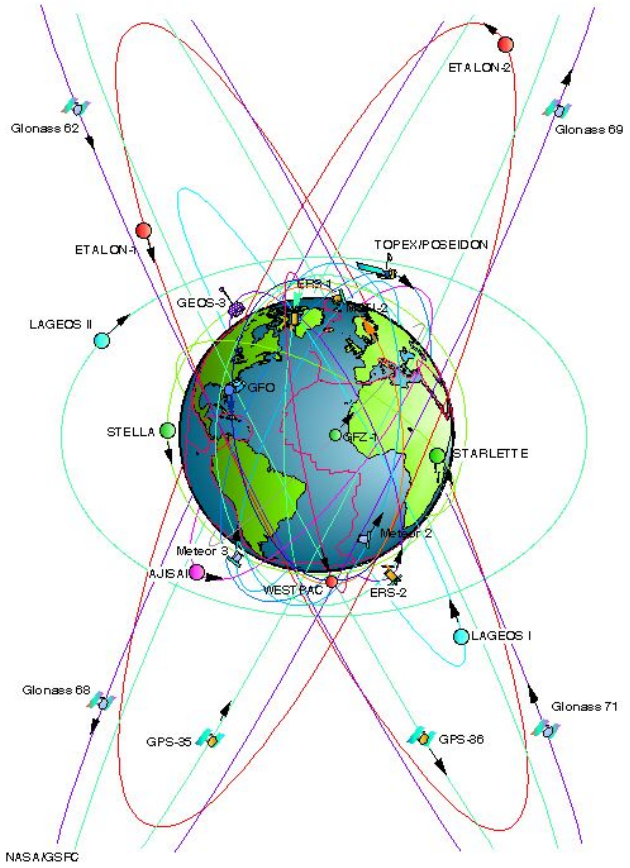
- Earth rotates in space once every 23 hr 56 m 04.09074 s (sidereal day)
- Earth rotates w.r.t. Sun every 24 hours (mean solar day)

- Earth rotation varies -- small ($< 10^{-9}$)
- It is stable because:
  - Angular momentum (~ large)
  - Equatorial bulge (~1/300)

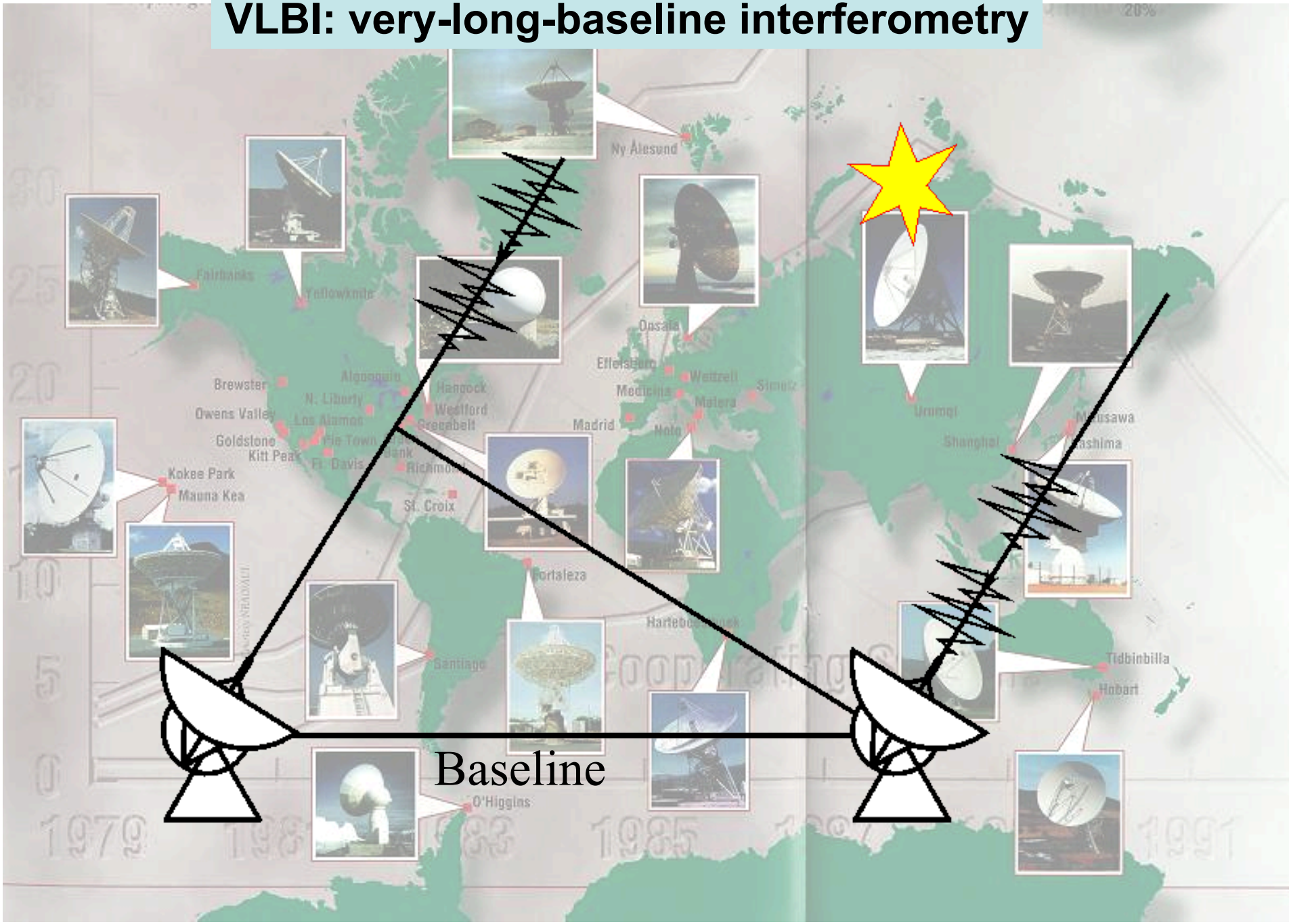


Satellites Laser Ranging

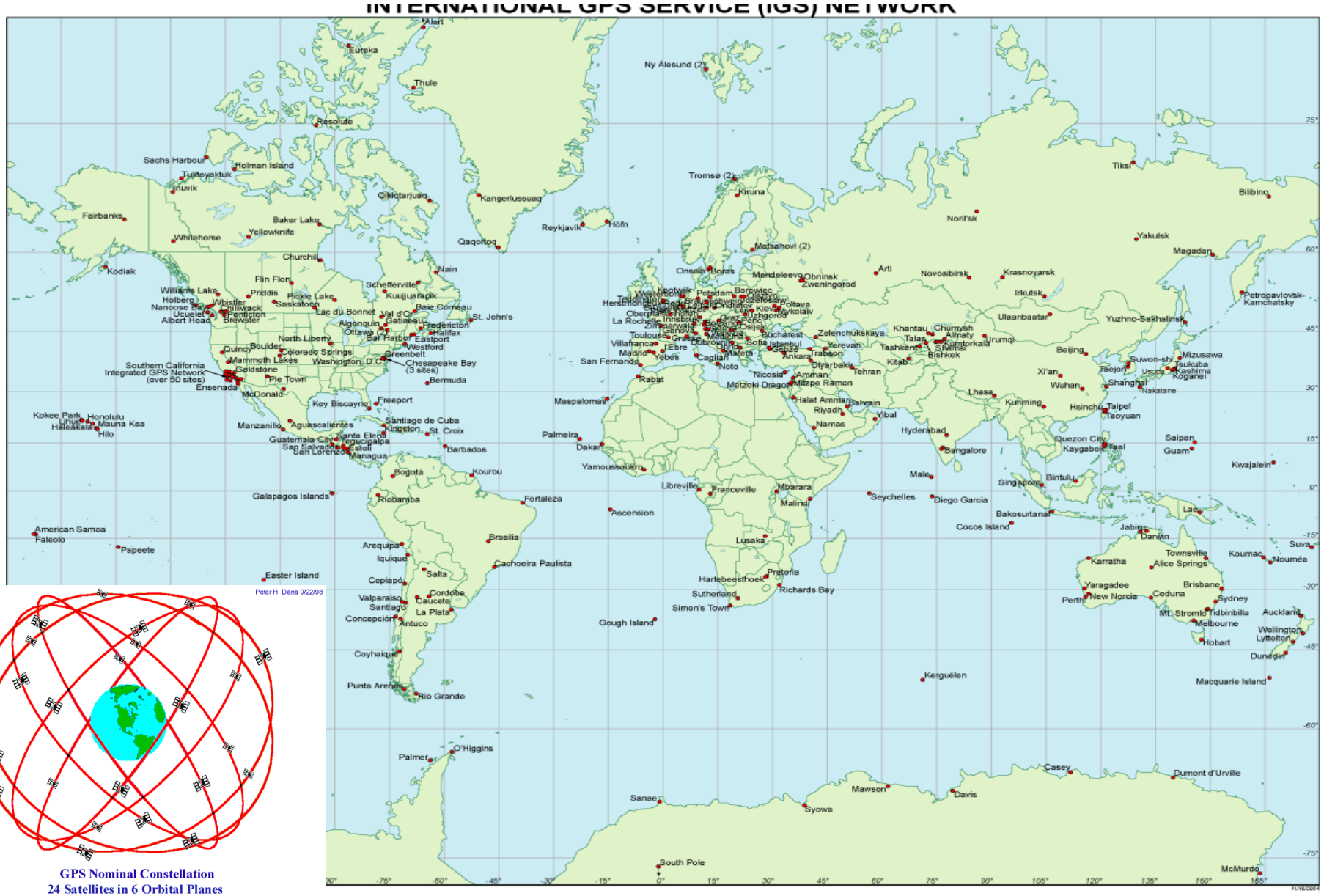




VLBI: very-long-baseline interferometry



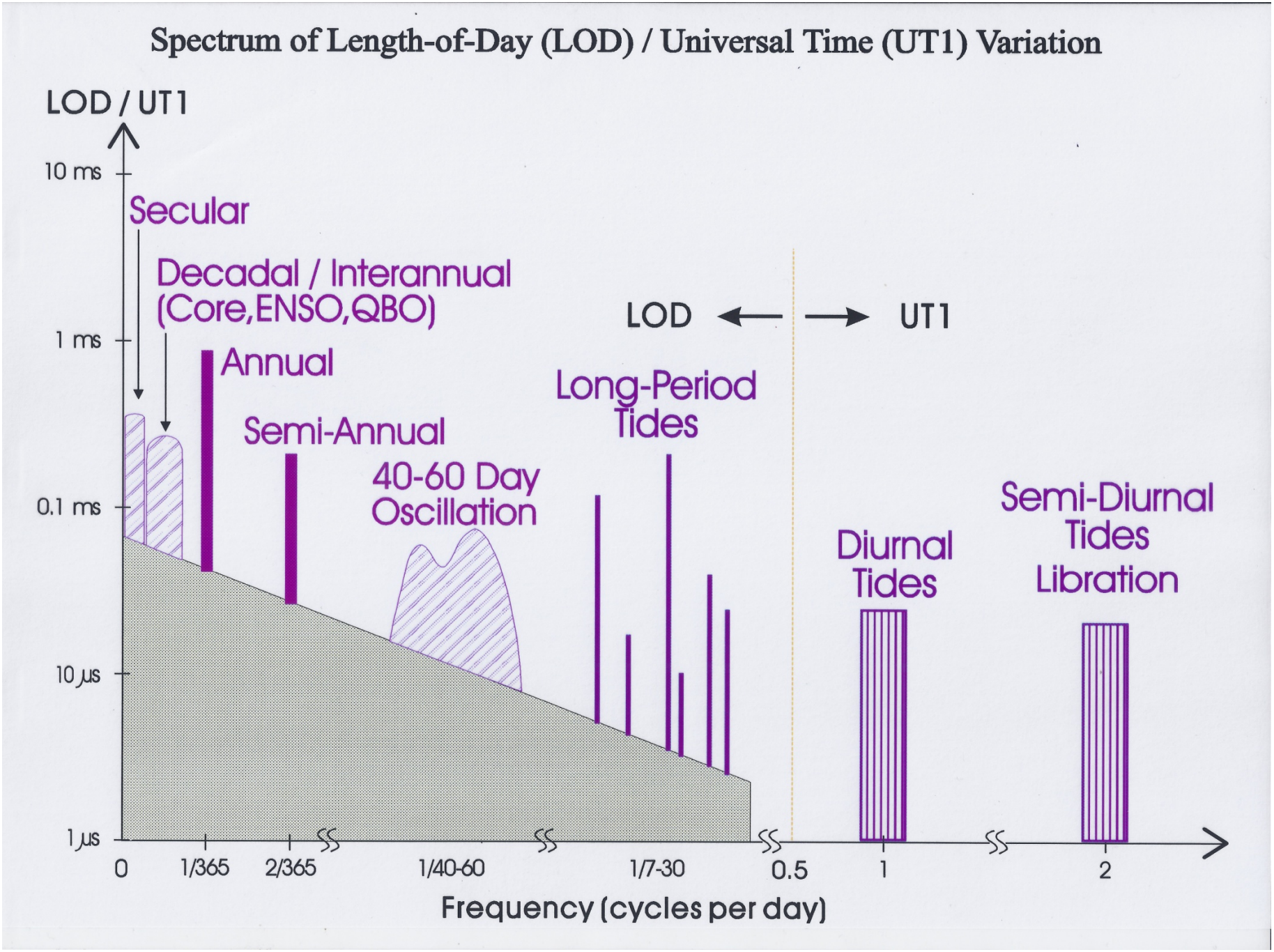
International GNSS Service network sites



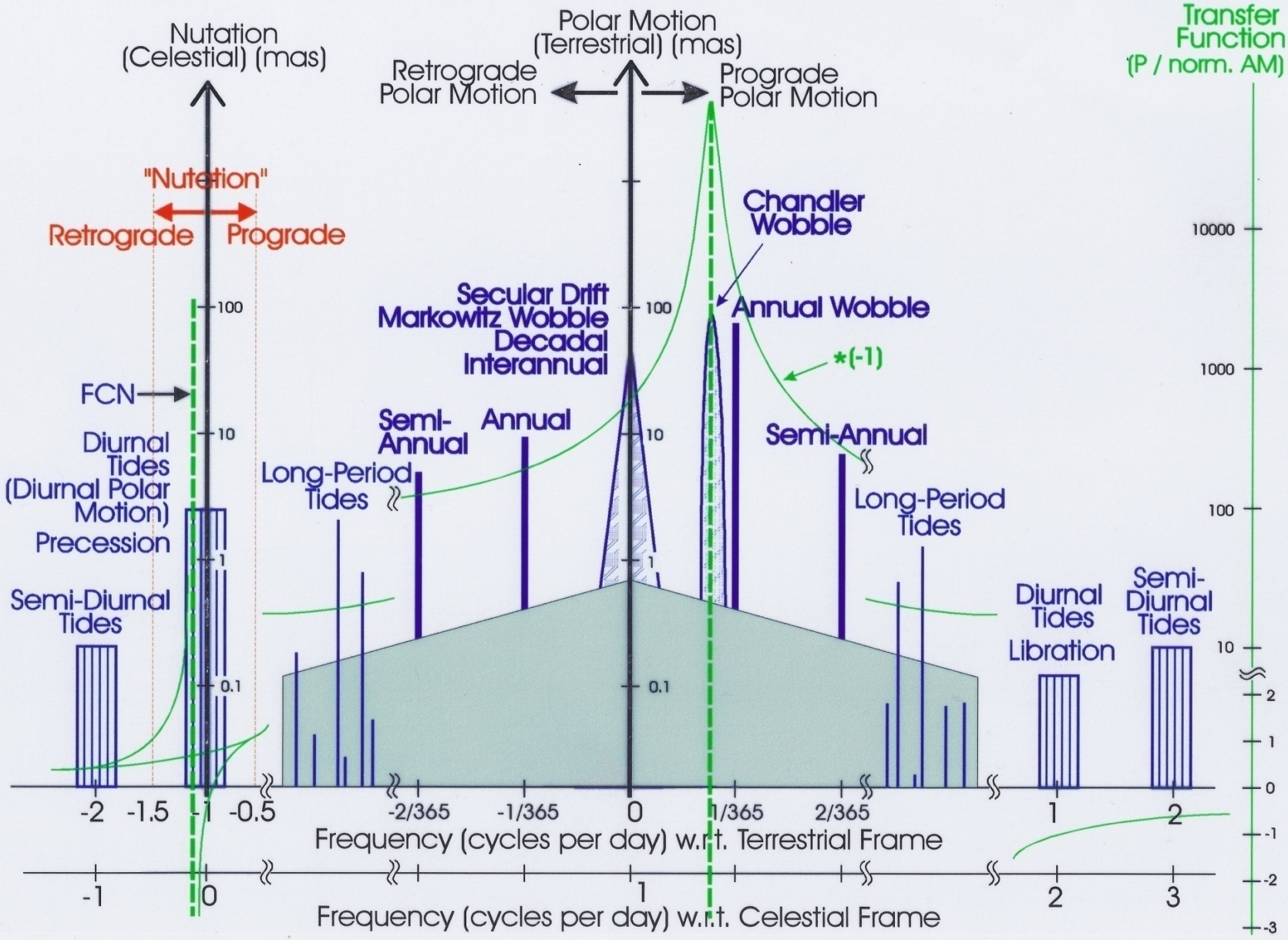
GPS Nominal Constellation
24 Satellites in 6 Orbital Planes
4 Satellites in each Plane
20,200 km Altitudes, 55 Degree Inclination

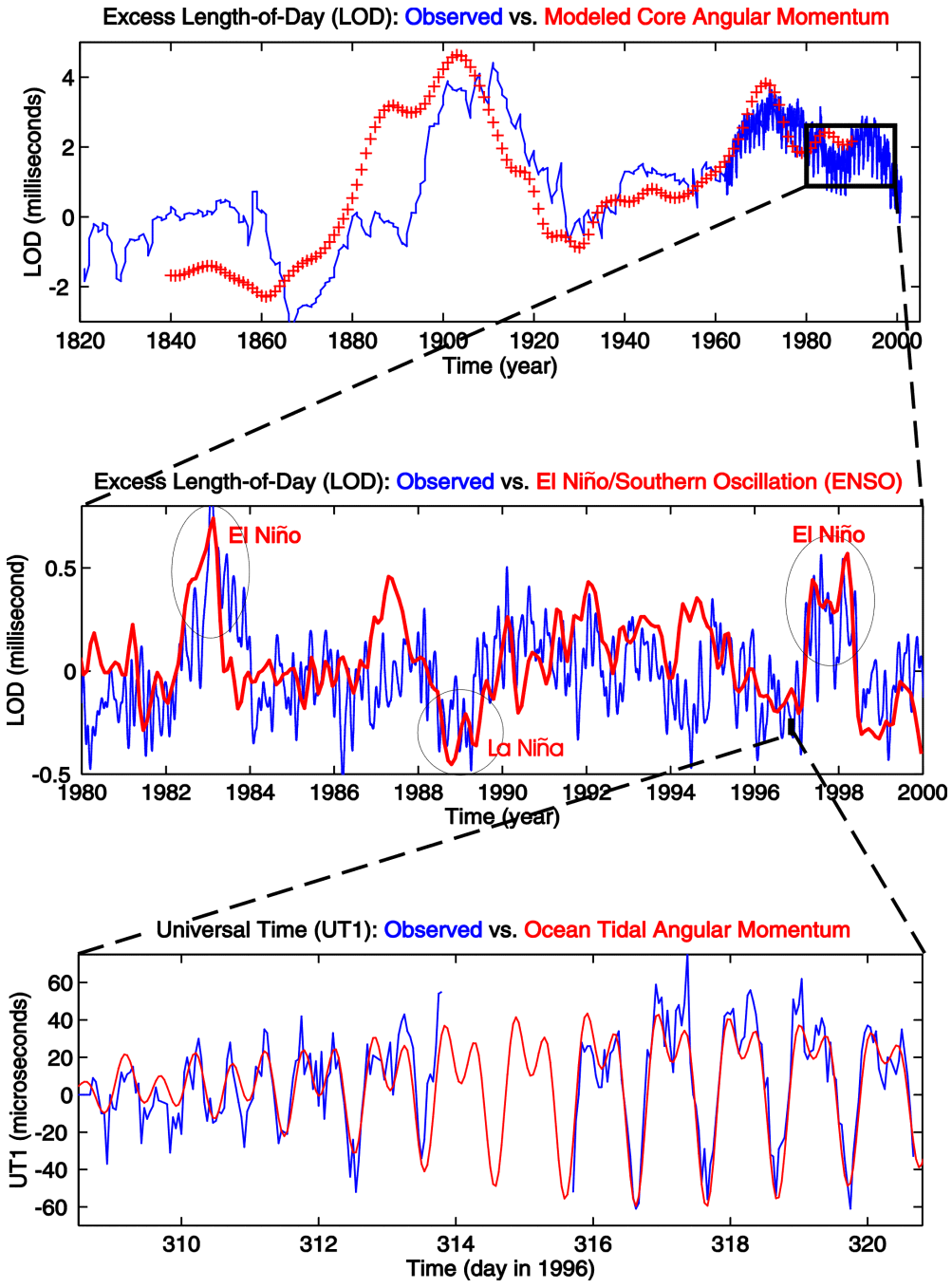
Earth's Rotation Variations, 3-D

- **Astronomical** (external torques → angular momentum changes)
  - Precession (進動), Nutations (章動), Librations (晃動)
  - Milankovitz cycles
  - Tidal braking (潮汐摩擦)
- **Geophysical** (internal forces → angular momentum exchanges among Earth components)
  - Length-of-day change (日長變化)
    - secular, long-term
    - decadal
    - interannual, seasonal, sub-seasonal, tidal
  - Polar motion (極移)
    - secular, long-term, seasonal, sub-seasonal, tidal
    - Chandler wobble (擺動), annual wobble
    - core nutations/wobbles

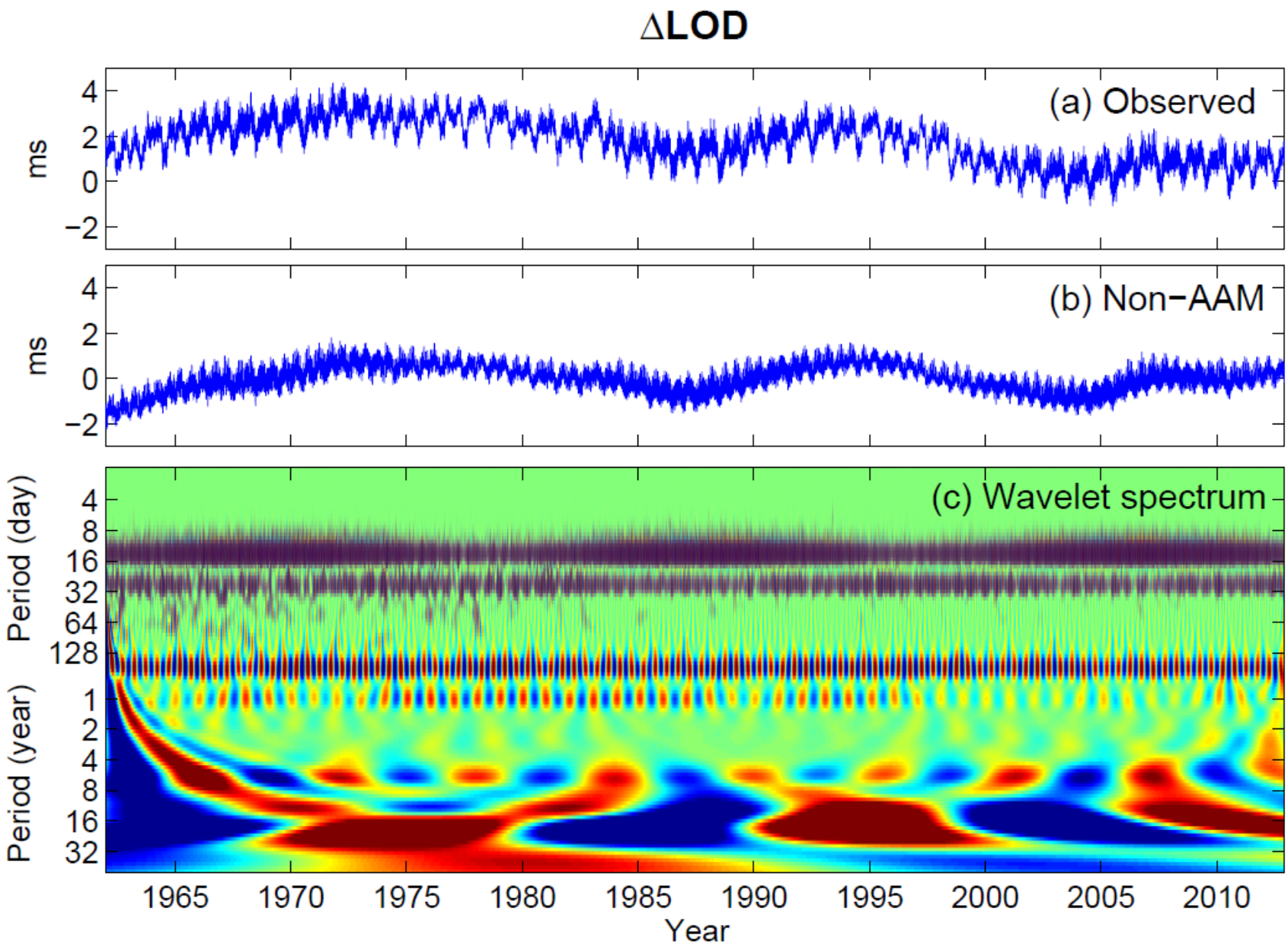

Spectrum of Length-of-Day (LOD) / Universal Time (UT1) Variation
LOD / UT1
10 ms
1 ms
0.1 ms
10 μs
1 μs
Secular
Decadal / Interannual
(Core,ENSO,QBO)
Annual
Semi-Annual
40-60 Day
Oscillation
LOD
UT1
Long-Period
Tides
Diurnal
Tides
Semi-Diurnal
Tides
Libration
0
1/365
2/365
1/40-60
1/7-30
0.5
1
2
Frequency (cycles per day)

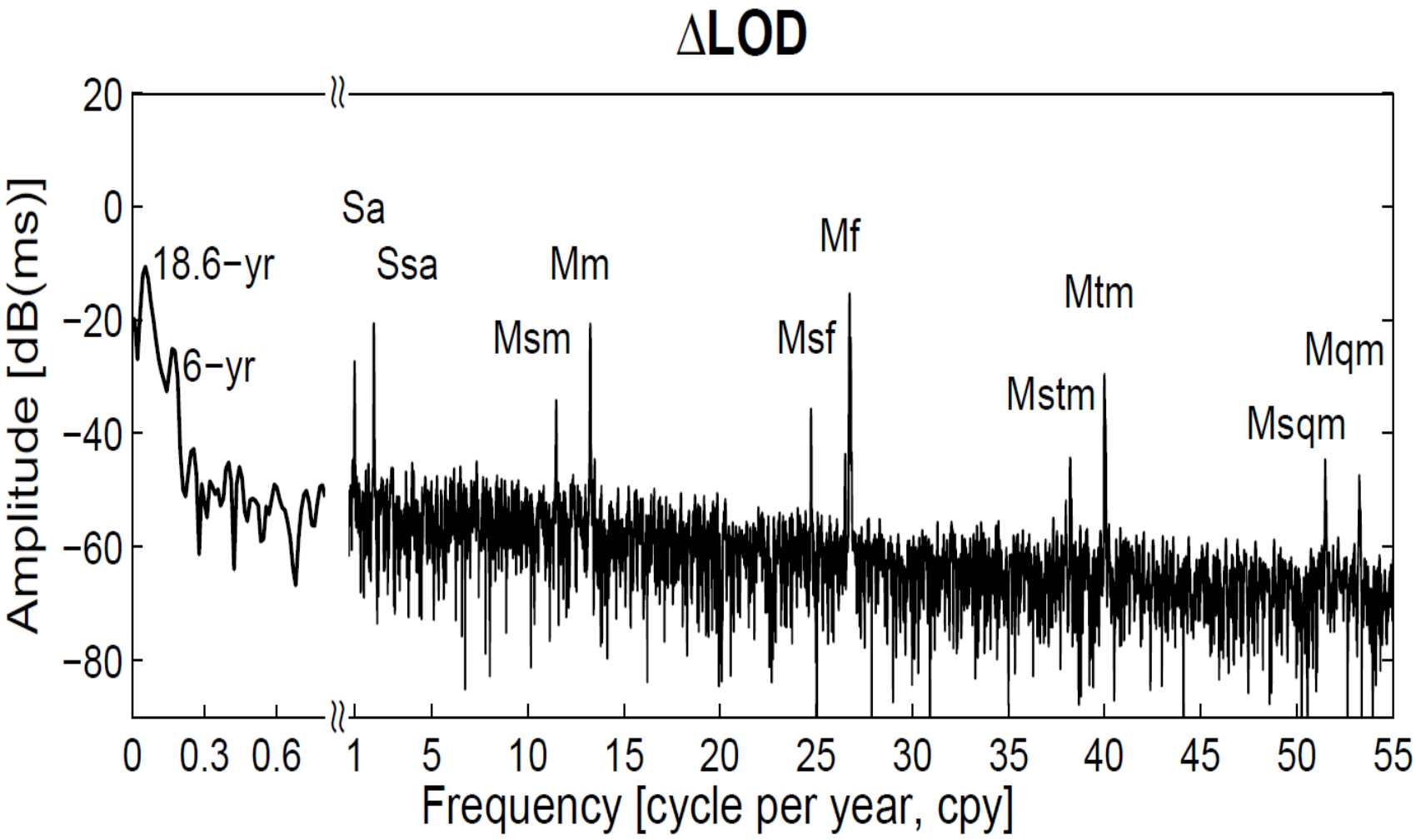
Spectrum of Polar Motion (P) / Nutation (N): $P = - N \exp(-i\phi)$

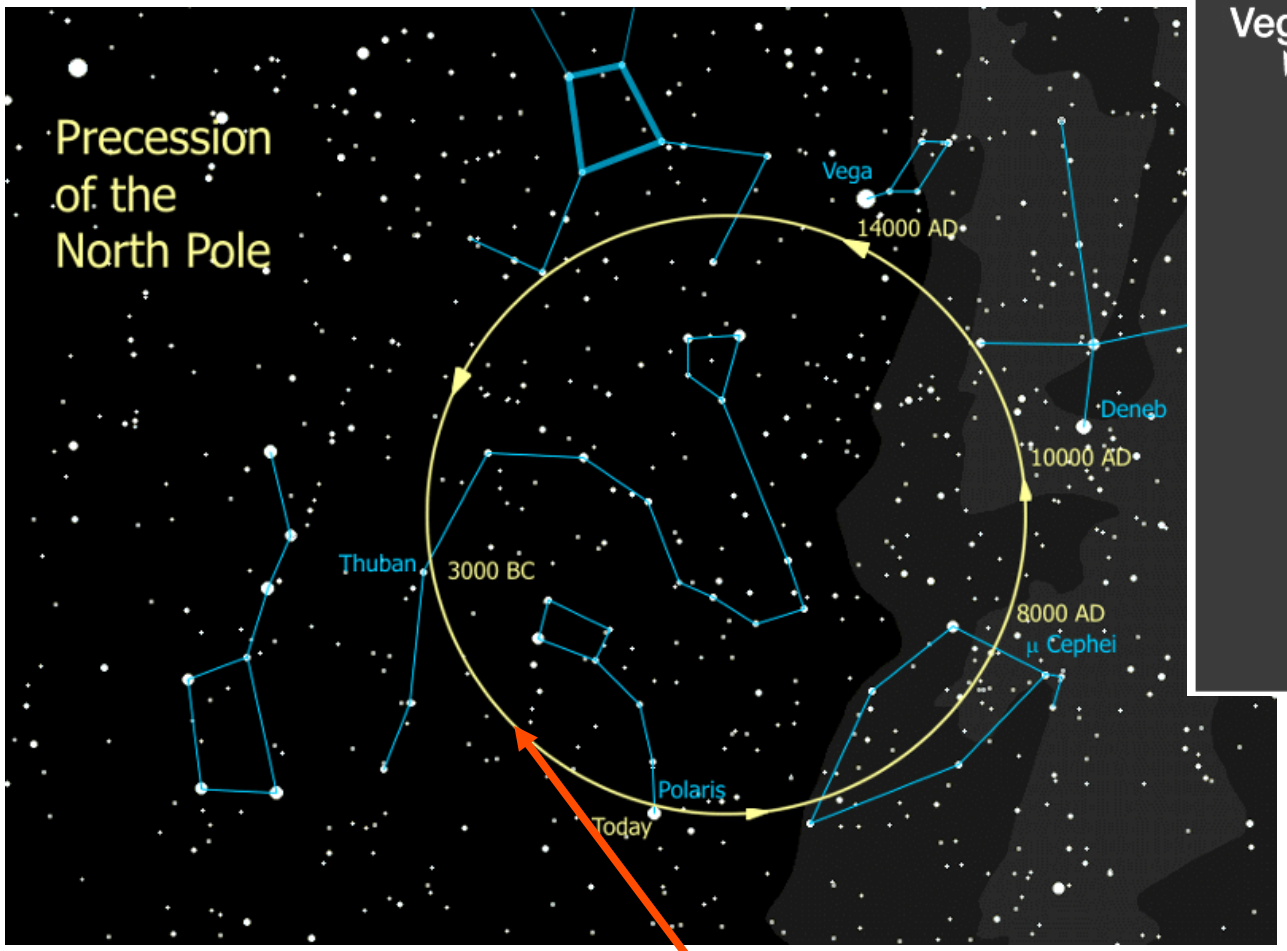


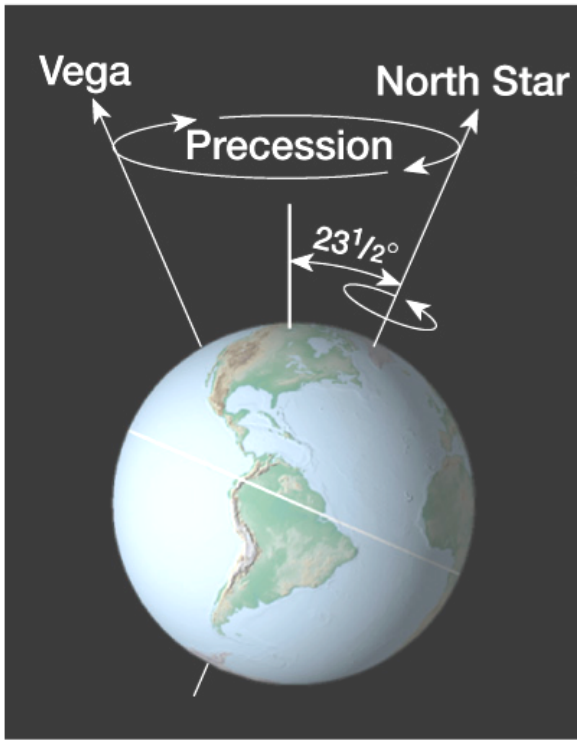

Excess Length-of-Day (LOD): Observed vs. Modeled Core Angular Momentum
LOD (milliseconds)
4
2
0
-2
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000
Time (year)
Excess Length-of-Day (LOD): Observed vs. El Niño/Southern Oscillation (ENSO)
El Niño
El Niño
La Niña
LOD (millisecond)
0.5
0
-0.5
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
Time (year)
Universal Time (UT1): Observed vs. Ocean Tidal Angular Momentum
UT1 (microseconds)
60
40
20
0
-20
-40
-60
310
312
314
316
318
320
Time (day in 1996)

(Chao, 2003)


ΔLOD
(a) Observed
ms
4
2
0
−2
(b) Non−AAM
ms
4
2
0
−2
(c) Wavelet spectrum
Period (day)
4
8
16
32
64
128
Period (year)
1
2
4
8
16
32
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
Year

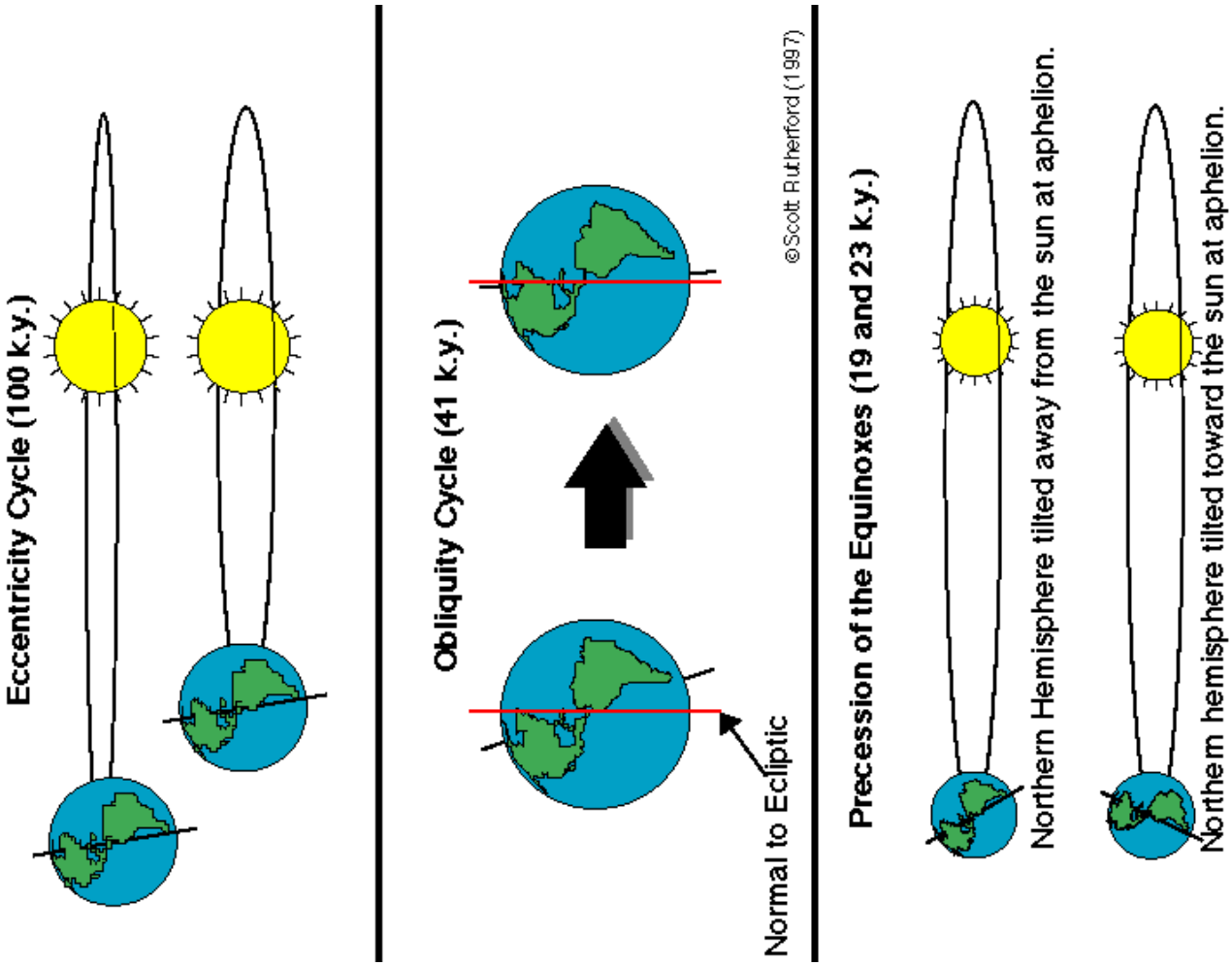

ΔLOD
Amplitude [dB(ms)]
20
0
−20
−40
−60
−80
18.6−yr
6−yr
Sa
Ssa
Mm
Msm
Mf
Msf
Mtm
Mstm
Mqm
Msqm
0
0.3
0.6
1
5
10
15
20
25
30
35
40
45
50
55
Frequency [cycle per year, cpy]



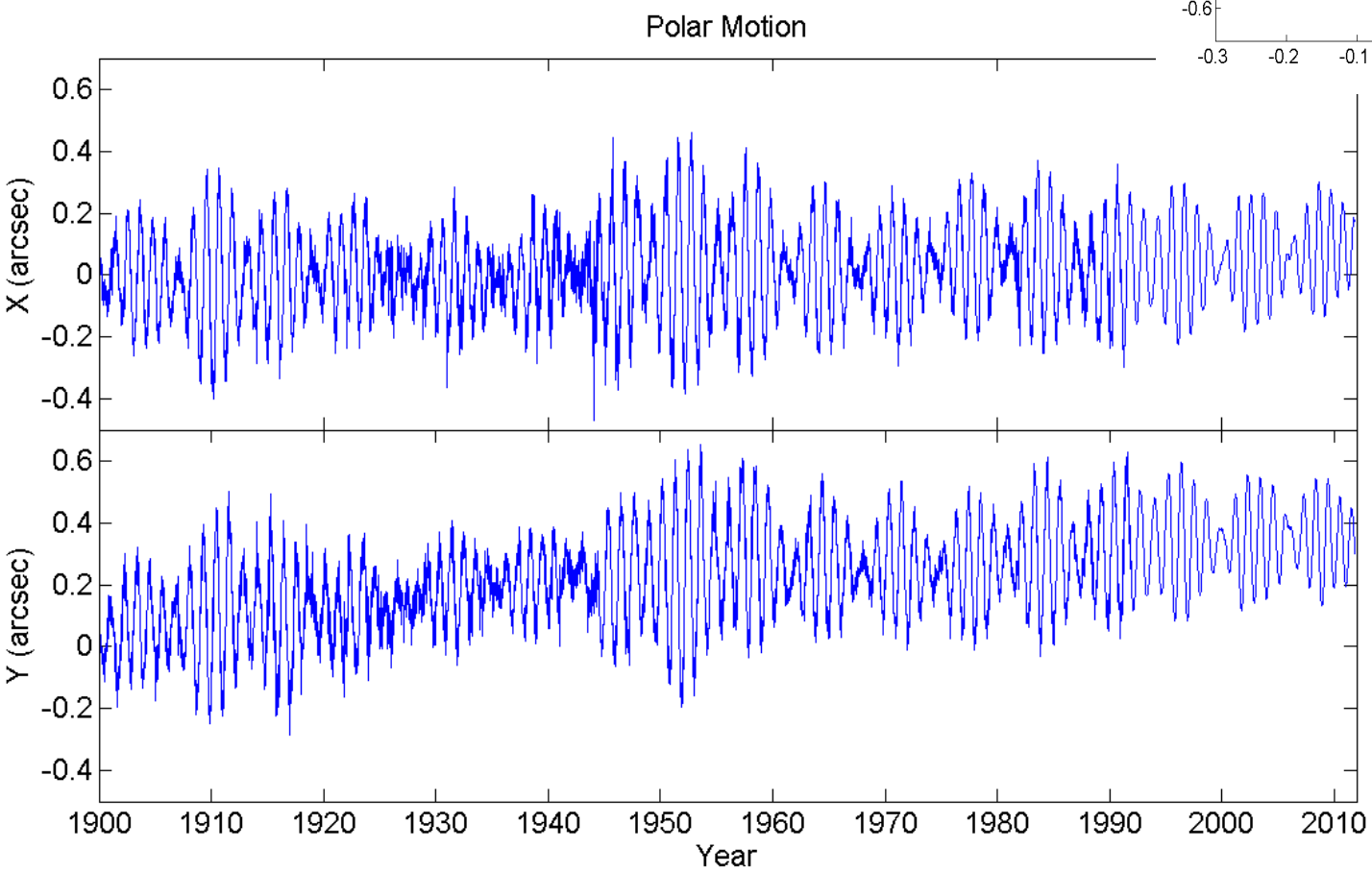


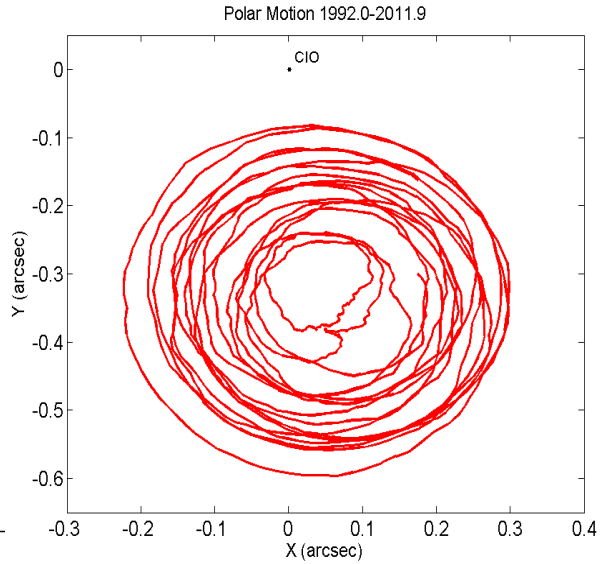
- 子曰：為政以德，譬如北辰，居其所，而眾星拱之。
- 《論語・為政》

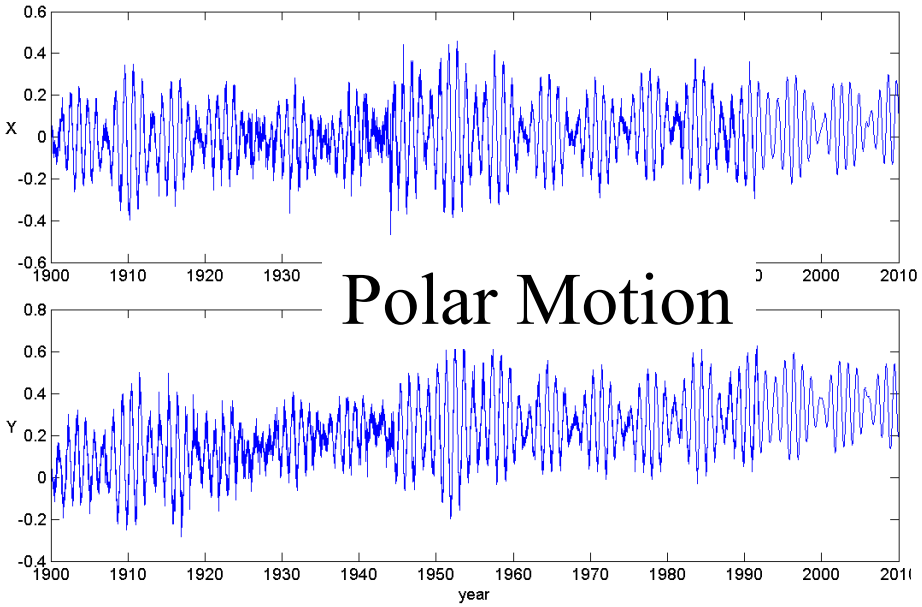
•Milankovitch Cycles



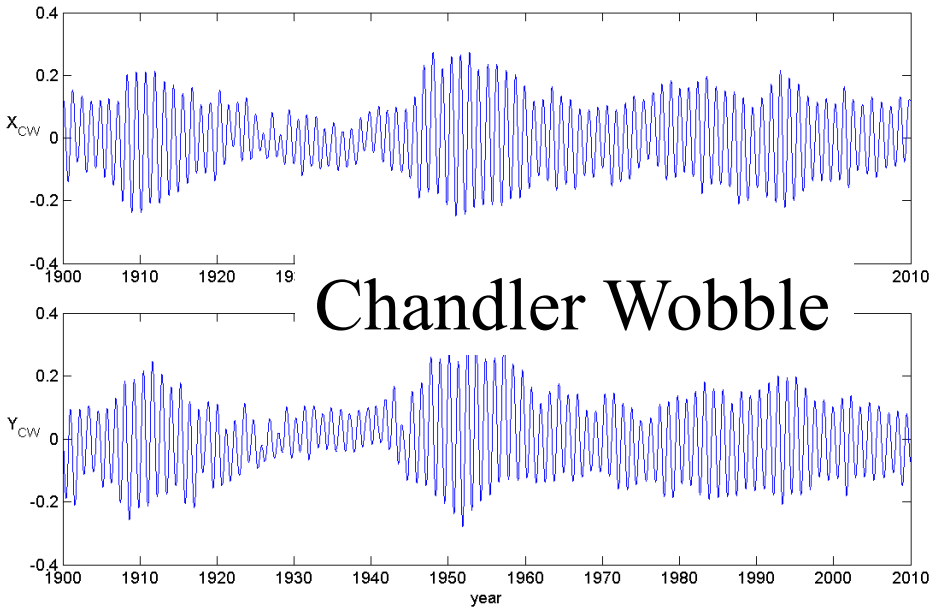
•“The Earth wobbles like a frisbee.”


Polar Motion
X (arcsec)
Y (arcsec)
Year
0.6
0.4
0.2
0
-0.2
-0.4
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010

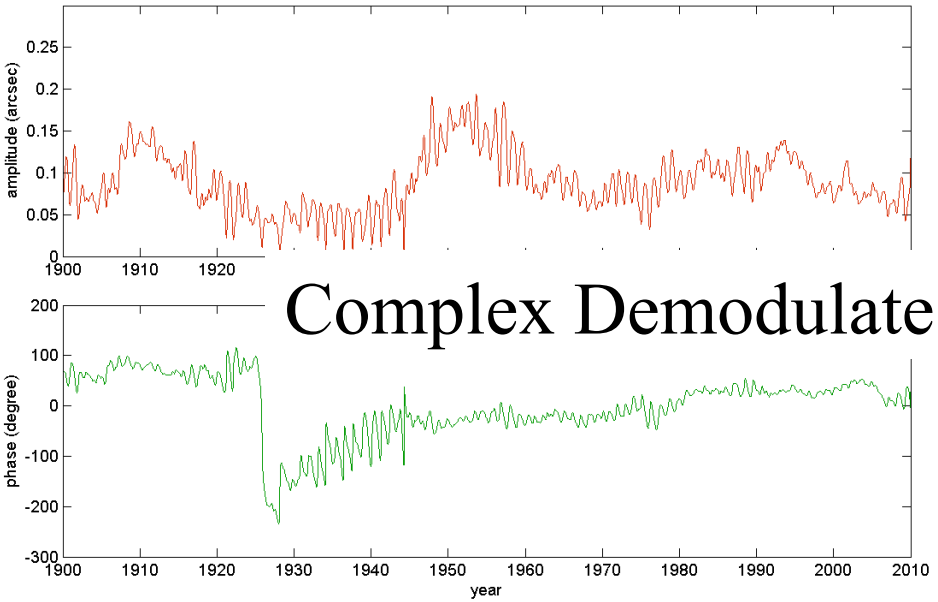

Polar Motion 1992.0-2011.9
CIO
Y (arcsec)
X (arcsec)
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4



Polar Motion



Chandler Wobble



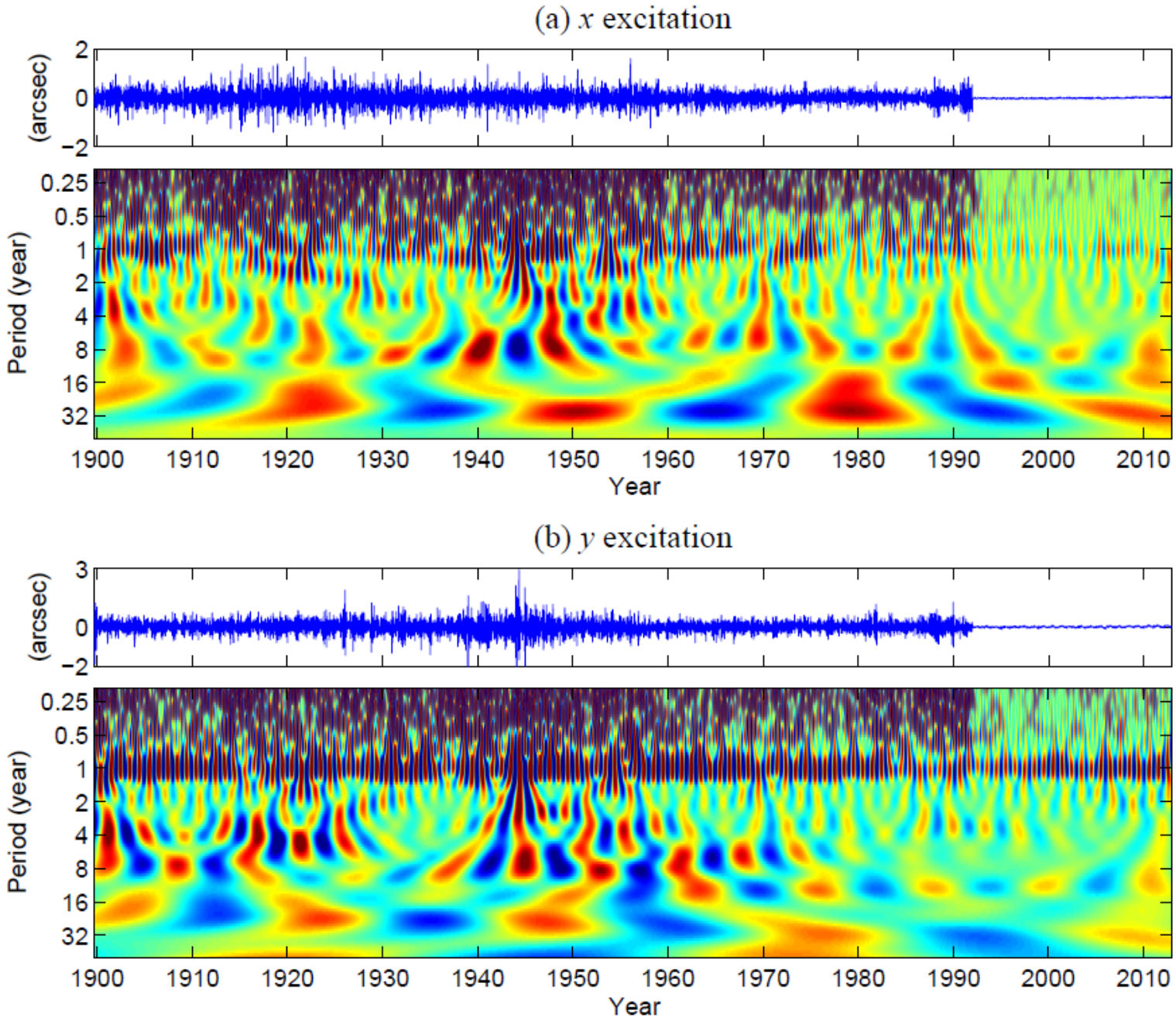
Complex Demodulate

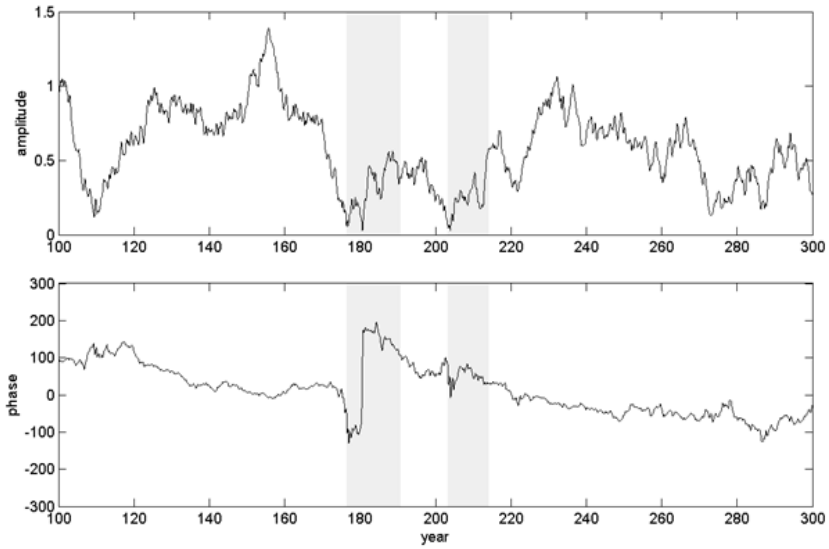
The polar motion excitation equation (linearized Liouville theory based on conservation of angular momentum, Munk & MacDonald, 1960):

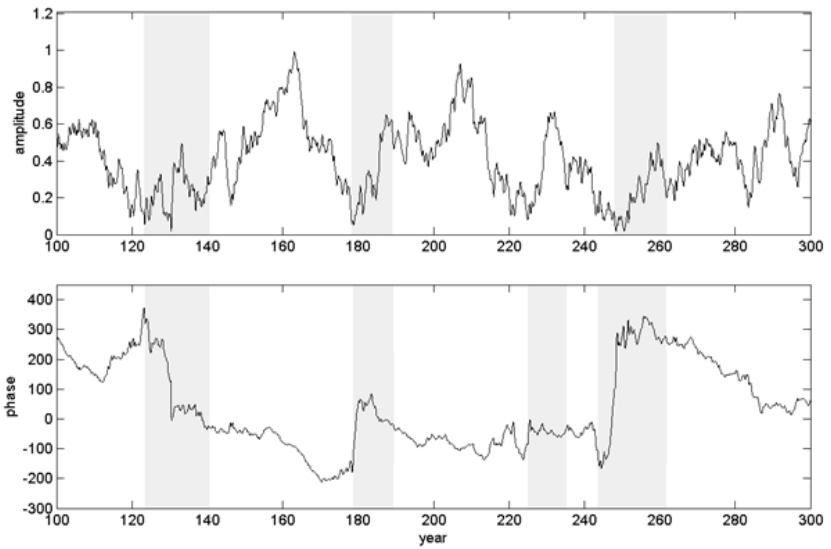
$$m(t) + i\, d/dt\, m(t) / \omega = E(t)$$

$$m(t) = -\, i\omega\, e^{i\omega(t\,-\tau)}\, E(\tau)\, d\tau\,.$$

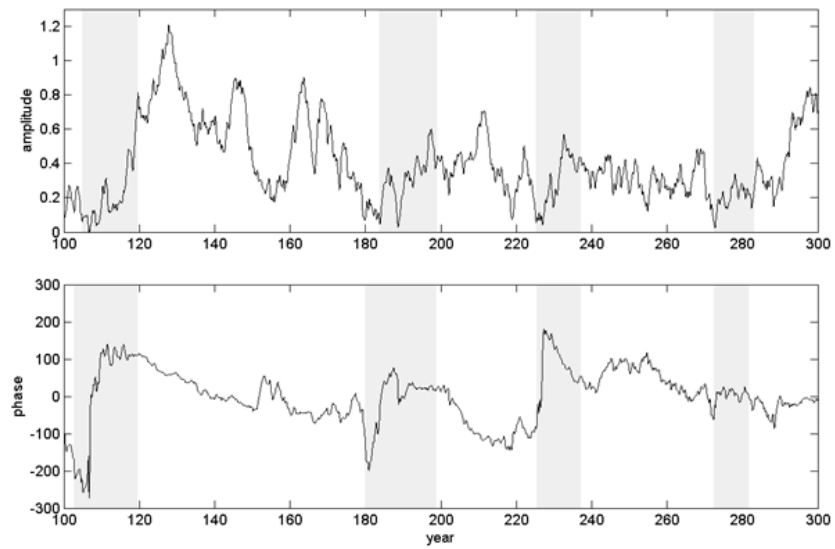
Note: This is a CONVOLUTION !!!


(a) x excitation
(arcsec)
2
0
−2
Period (year)
0.25
0.5
1
2
4
8
16
32
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Year
(b) y excitation
(arcsec)
3
0
−2
Period (year)
0.25
0.5
1
2
4
8
16
32
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Year


amplitude
phase
year


amplitude
phase
year


amplitude
phase
year


amplitude
phase
year

Measuring Gravity

- Surface
  - centrifugal (pseudo-)force correction
  - free-air anomaly
  - Bouguer anomaly
- Air-borne, Ship-borne
  - centrifugal (pseudo-)force correction
  - Coriolis (pseudo-)force correction (Eötvös effect)
  - platform acceleration (“noise”)
- Space-born
  - gravity canceled by centrifugal force (free fall)

⇨ need orbit perturbations, hence “tracking”

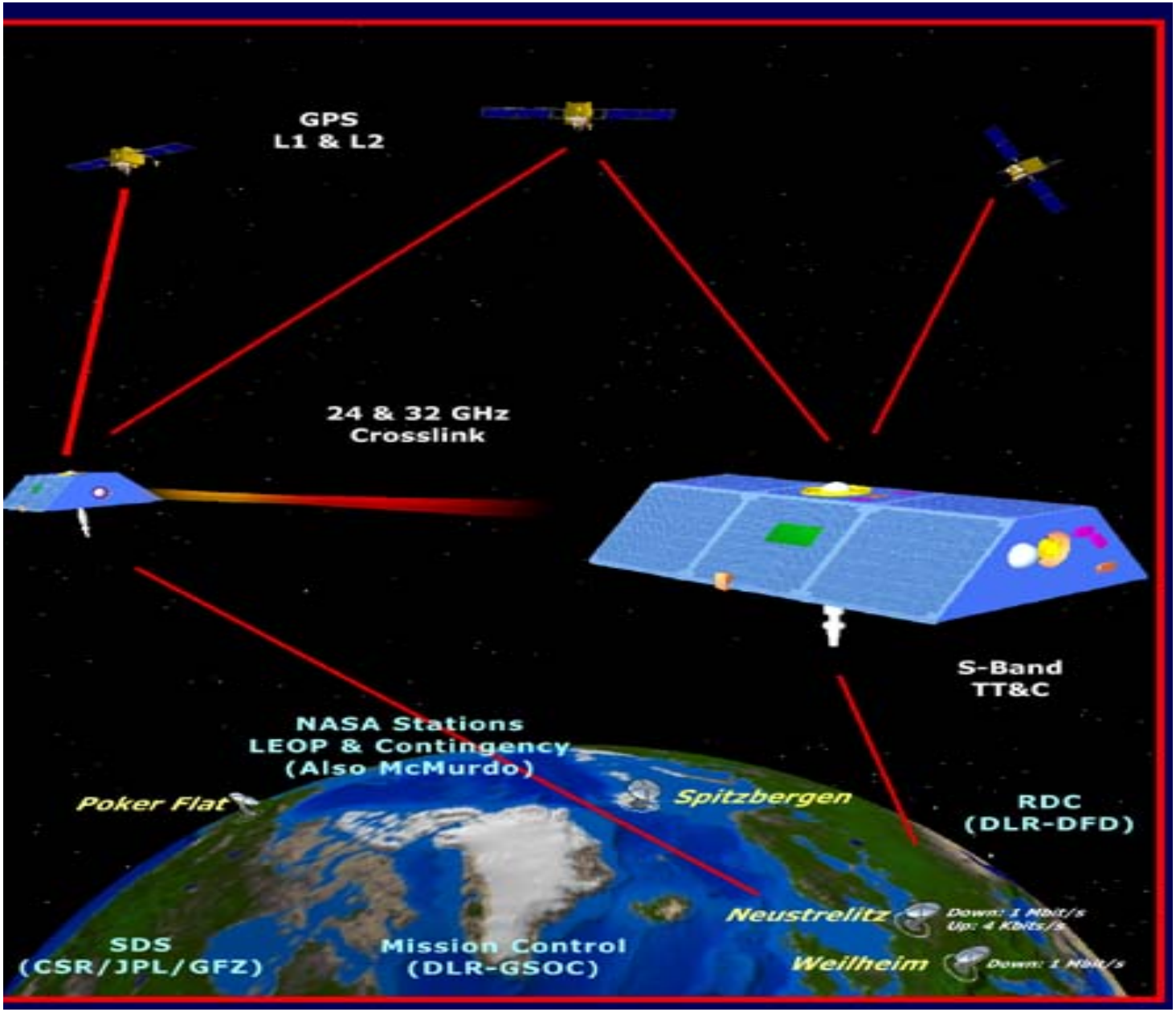
"Measuring" Gravity from Space : Milestones

- 1957 Sputnik II – J_2 (Earth oblateness)
- Kaula's (1966) theory (spherical harmonics)
- Early 1970s, Satellite-laser-ranging
- Spherical harmonic solutions
  - ~1970s, 10 x 10
  - ~1980s, 36 x 36, 70 x 70
  - ~1990s, 180 x 180, 360 x 360
  - "EGM 2008", degree/order 2159 (!)
- time-variable: "J_2 dot" (1980s)
- time-variable: J_2 and low-degree variations (1990s)
- CHAMP (2000), GRACE (2002), GOCE (2009)

Time-Variable Gravity

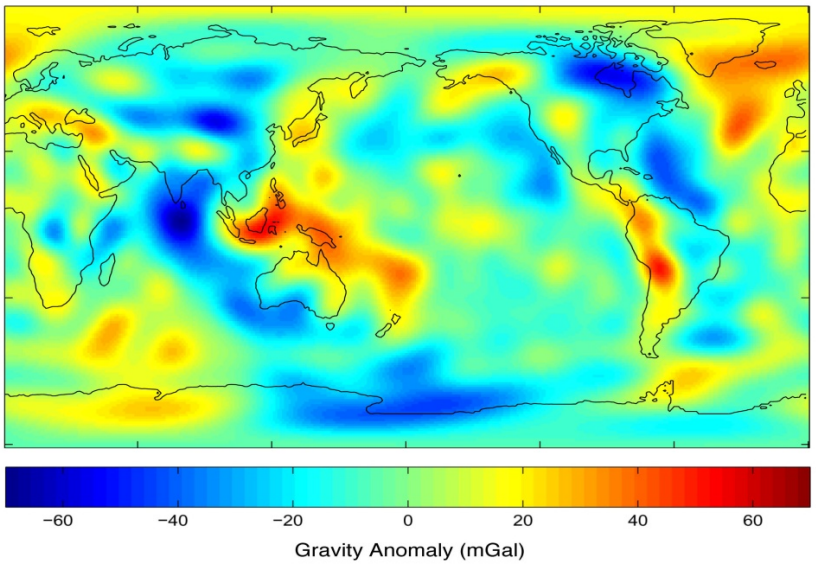
- atmospheric circulation
- tides
- **land hydrology (seasonal + anomaly)**
- **cryospheric mass (ice sheets, glaciers, permafrost)**
- **glacial isostatic adjustment GIA / post-glacial rebound PGR**
- **ocean circulations, ENSO, sea level …**
- **largest earthquakes**
- tectonics
- core flow
- …

GRACE (Gravity Recovery And Climate Experiment)

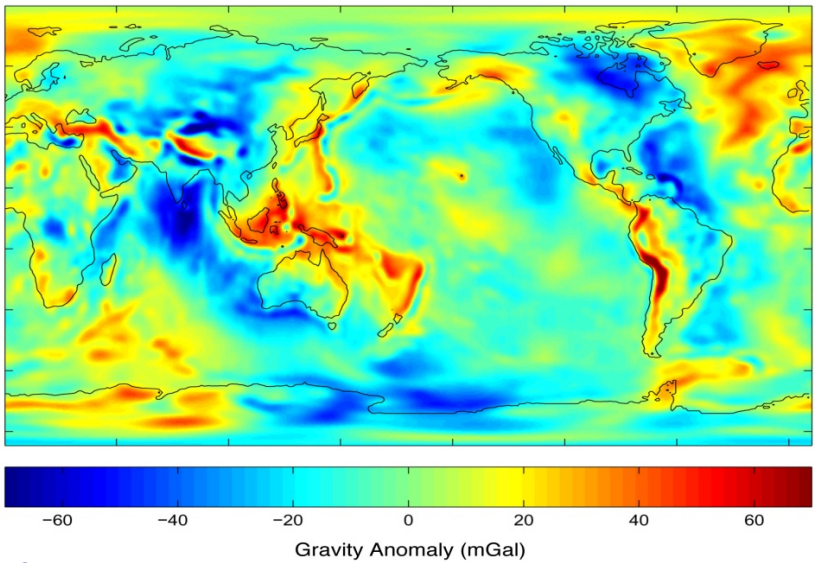


Static Gravity Anomaly

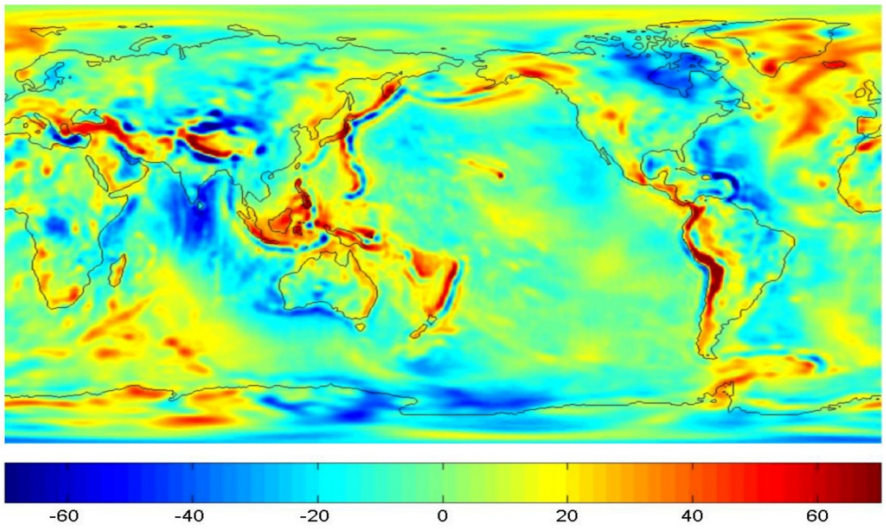
4 Decades of tracking to geodetic satellites



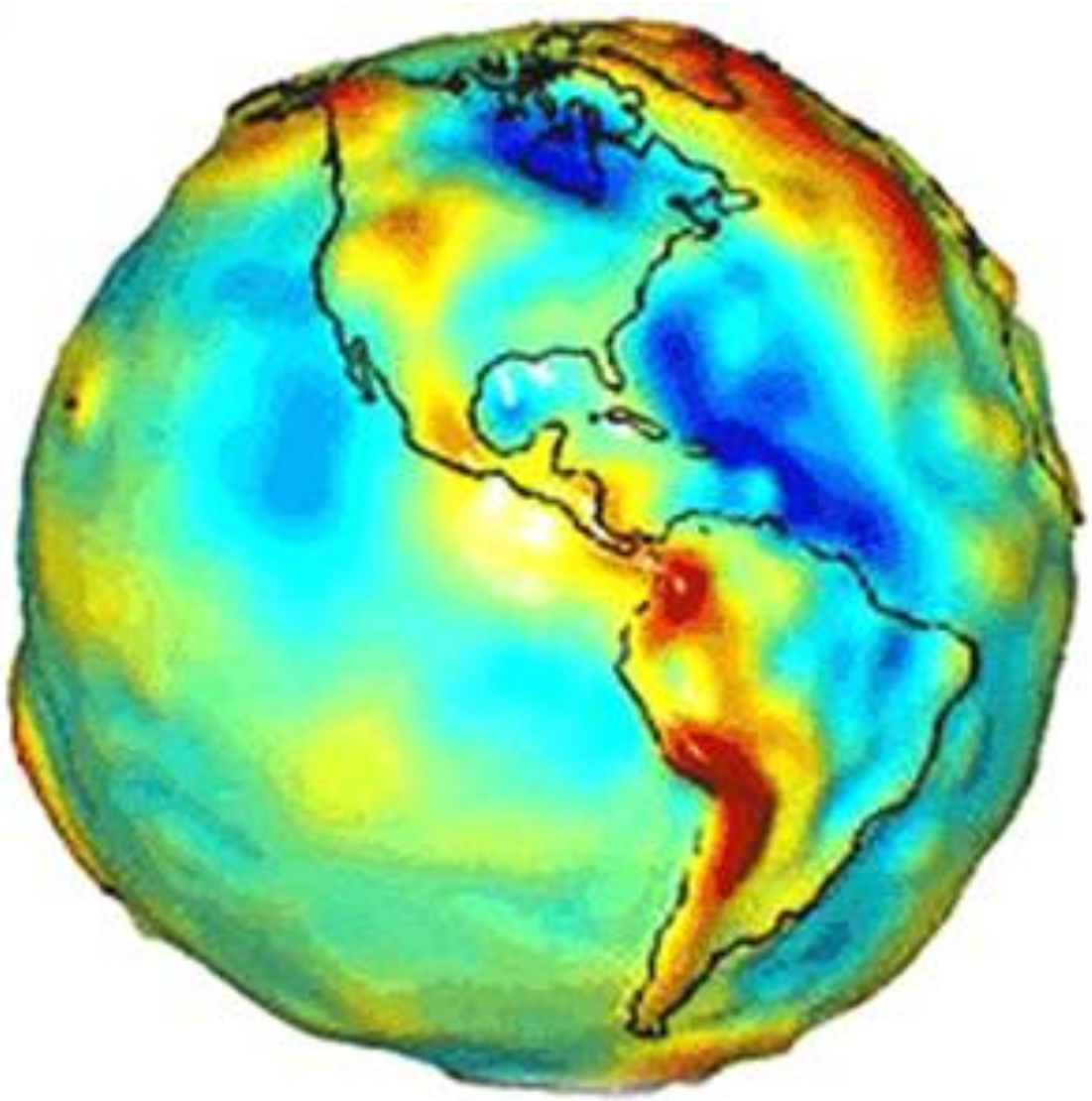
111 days of GRACE data

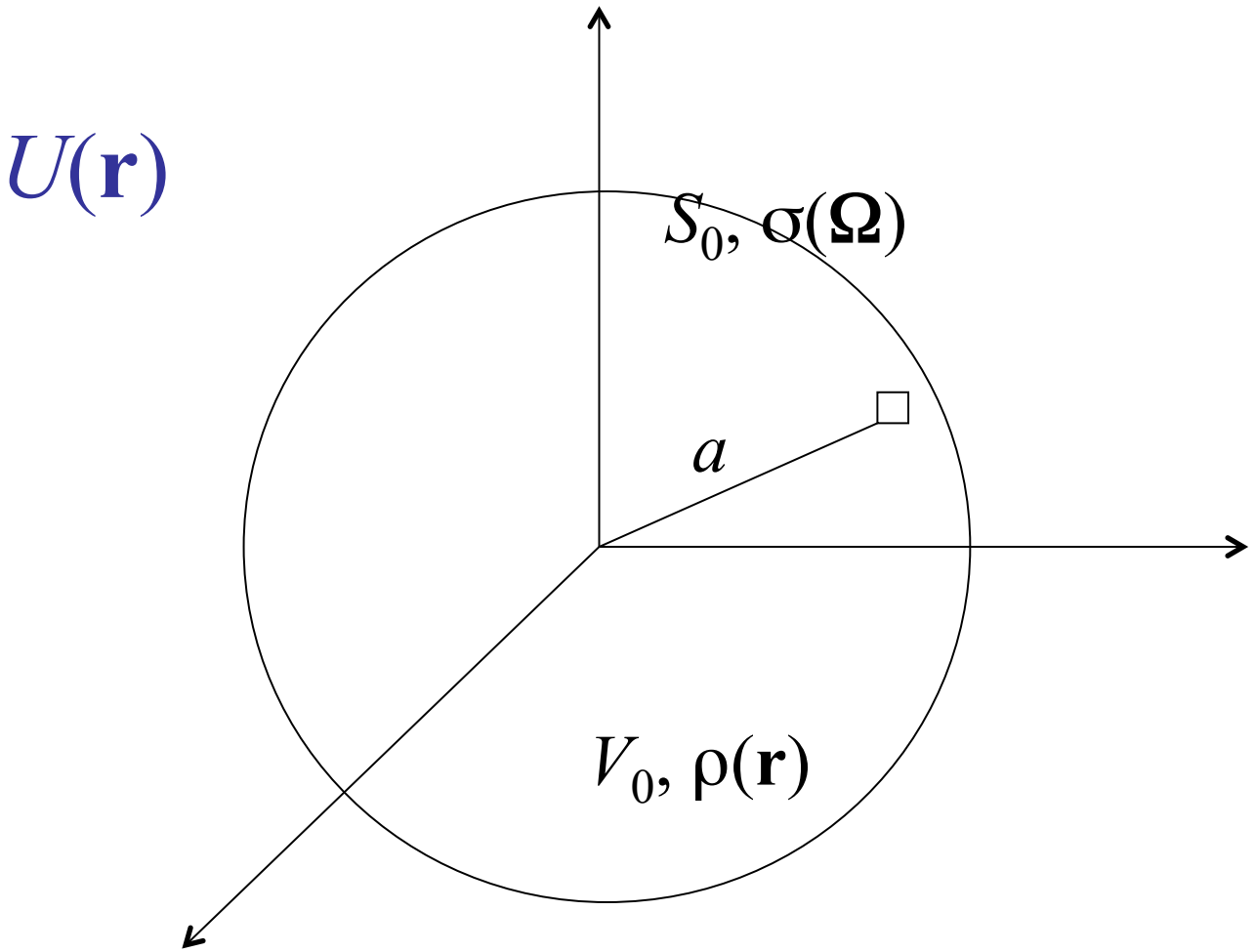


13 months of GRACE data



Earth's Geoid (Static)




$U(\mathbf{r})$
S_0, $\sigma(\mathbf{\Omega})$
a
V_0, $\rho(\mathbf{r})$

Gravitational Potential Field

- Newton's gravitational law

$$U(\mathbf{r}) = G \iiint_{V_0} \frac{\rho(\mathbf{r}_0)}{|\,\mathbf{r} - \mathbf{r}_0\,|} dV_0$$

$$1/|\,\mathbf{r} - \mathbf{r}_0\,| = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} (2n+1)^{-1} (r_0^{\,n} / r^{n+1}) Y_{nm}^{\,*}(\Omega) Y_{nm}(\Omega_0)$$

⇒ Multipole expansion of gravity field

$$U(\mathbf{r}) = G \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{1}{(2n+1) r^{n+1}} \left[\iiint_{V_0} \rho(\mathbf{r}_0)\, r_0^{\,n}\, Y_{nm}(\Omega_0)\, dV_0 \right] Y_{nm}^{\,*}(\Omega)$$

Gravitational Potential Field (Geoid)

- Multipole expansion of Newton's formula:

$$U(\mathbf{r}) = G \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{1}{(2n+1)\, r^{n+1}} \left[\iiint_{V_0} \rho(\mathbf{r}_0)\, r_0^{\,n}\, Y_{nm}(\Omega_0)\, dV_0 \right] Y_{nm}^{\,*}(\Omega)$$

- Conventional expression (satisfying Laplace Eq. in terms of Stokes Coeff.):

$$U(r,\theta,\lambda) = \frac{GM}{a} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r} \right)^{n+1} P_{nm}(\cos\theta) \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right)$$

$$\Rightarrow \quad \boxed{C_{nm} + iS_{nm} = \frac{1}{(2n+1)\, M a^n} \iiint_{V_0} \rho(\mathbf{r})\, r^n\, Y_{nm}(\Omega)\, dV}$$

3-D Gravitational Inversion

- Multipole expansion

$$U(\mathbf{r}) = G \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \frac{1}{(2n+1)\, r^{n+1}} \left[\int\int_{V_0}\int \rho(\mathbf{r}_0)\, r_0^{\,n}\, Y_{nm}(\Omega_0)\, dV_0 \right] Y_{nm}^{\,*}(\Omega)$$

$2n+1$ (known) multipoles for each degree n

- Moment expansion

$$U(\mathbf{r}) = G \sum_{n=0}^{\infty} \sum_{\alpha,\beta,\gamma \geq 0}^{\alpha+\beta+\gamma=n} \frac{(-1)^n}{\alpha!\,\beta!\,\gamma!} \left[\int\int_{V_0}\int x_0^{\alpha}\, y_0^{\beta}\, z_0^{\gamma}\, \rho(\mathbf{r}_0)\, dV_0 \right] \frac{\partial^n}{\partial x^{\alpha}\, \partial y^{\beta}\, \partial z^{\gamma}} \left(\frac{1}{|\mathbf{r}|} \right)$$

$(n+1)(n+2)/2$ (unknown) moments for each n

⇒ "Degree of deficiency" of knowledge is $n(n-1)/2$ for each degree n.

Degree n	# multipoles $(2n + 1)$	# moments $(n+1)(n+2)/2$	Degree of deficiency $n(n-1)/2$
0	1 (monopole)	1 (total mass)	0
1	3 (dipole)	3 (center of mass)	0
2	5 (quadrupole)	6 (inertia tensor)	1
3	7 (octupole)	10 (3rd moment)	3
4	9	15	6
5	11	21	10
6	13	28	15
100	201	5151	4950

The degree of deficiency as a function of spherical harmonic degree n in the 3-D gravitational inversion.

Additional physical/mathematical constraints leading to unique solutions:

- minimum shear energy
- maximum entropy of ρ
- minimum L-2 norm variance for the lateral distribution
- ……

2-D gravitational Inversion on a spherical shell S_0

$$C_{nm} + iS_{nm} = \frac{a^2}{(2n+1)M} \iint_{S_0} \sigma(\Omega)\, Y_{nm}(\Omega)\, d\Omega$$

$$\sigma(\Omega) = \Sigma_{n,m} \sigma_{nm} Y^*_{nm}(\Omega)$$

$$\Rightarrow \quad \sigma_{nm} = \frac{(2n+1)M}{4\pi a^2} \left(C_{nm} + iS_{nm} \right)$$

It is possible to "generate" ANY external field by means of some proper surface density on S_0.

2-D gravitational Inversion on a spherical shell S_0 (cont'd)

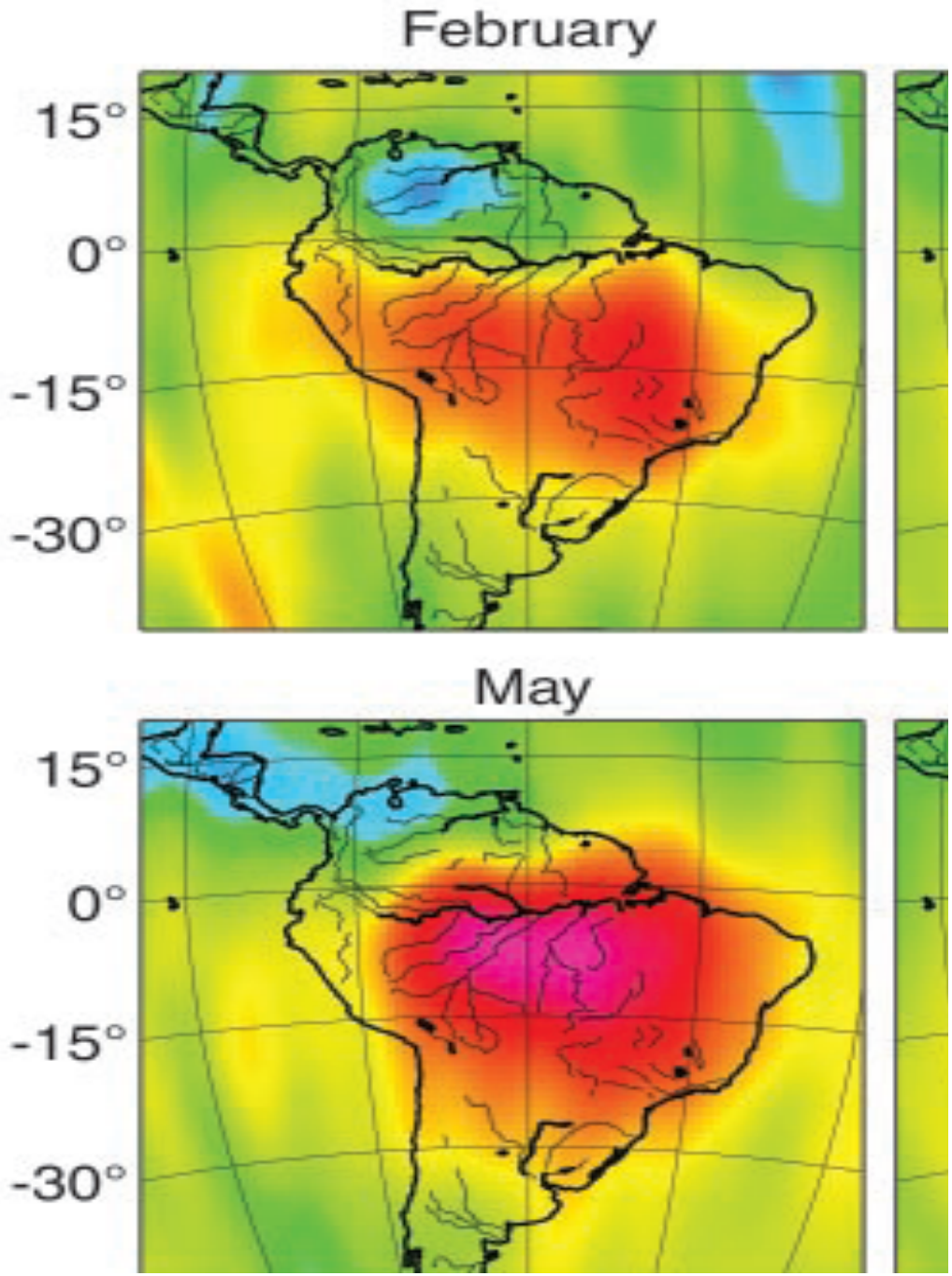
For CHANGES due to mass redistribution on S_0 (taking into account of loading effect), in Eulerian description:

$$\Delta C_{nm}(t) + i\,\Delta S_{nm}(t) = \frac{a^2(1+k'_n)}{(2n+1)M}\int\int \Delta\sigma(\Omega;t)\, Y_{nm}(\Omega)\, d\Omega$$

$$\Rightarrow \quad \Delta\sigma_{nm}(t) = \frac{(2n+1)M}{4\pi a^2(1+k'_n)}\left[\Delta C_{nm}(t) + i\,\Delta S_{nm}(t)\right]$$

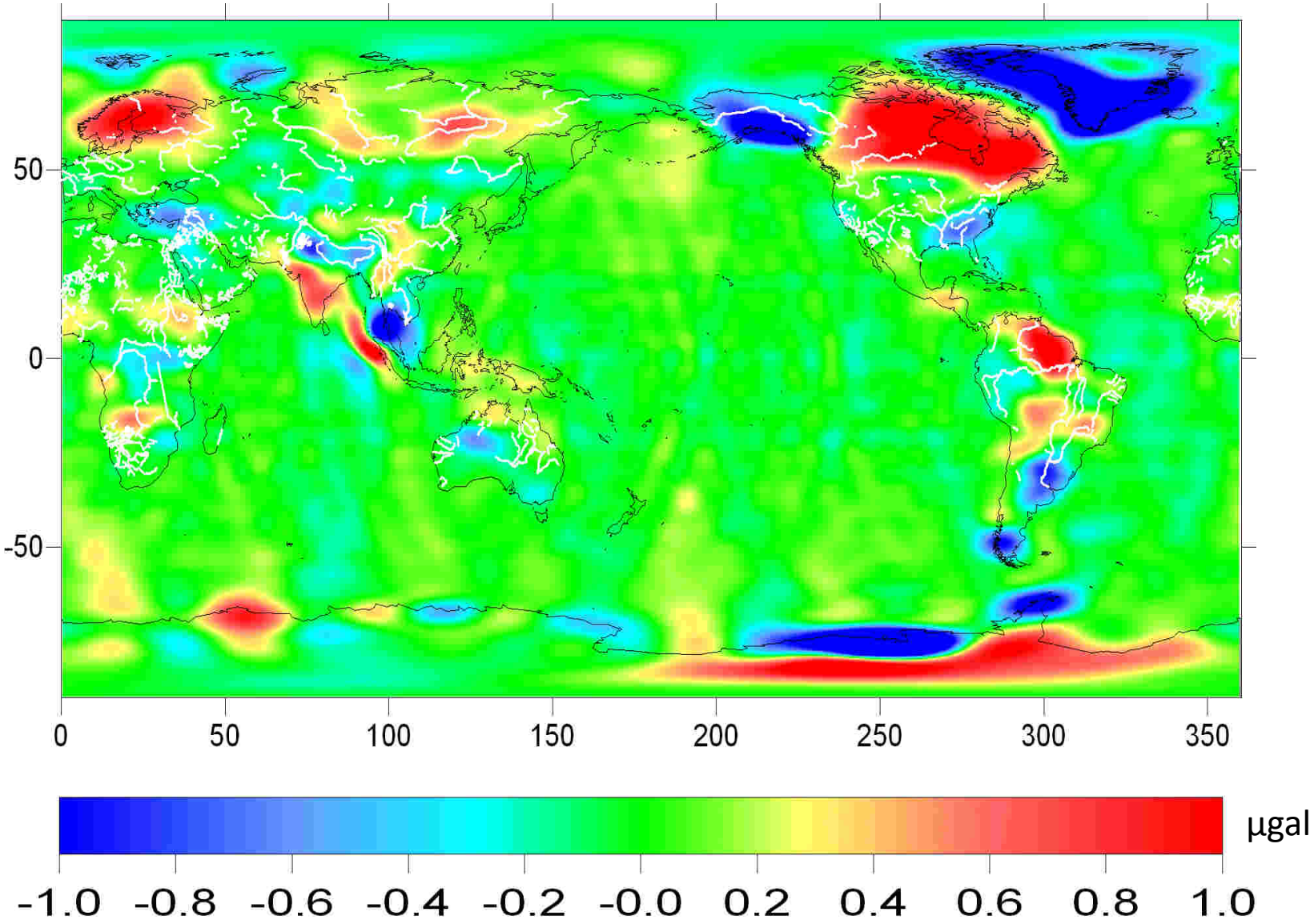
Unique!

Loading effect "undone"



GRACE, 2003
(Tapley et al, 2004)

Trend of global gravity anomaly changes from GRACE (by fan filter 300km)



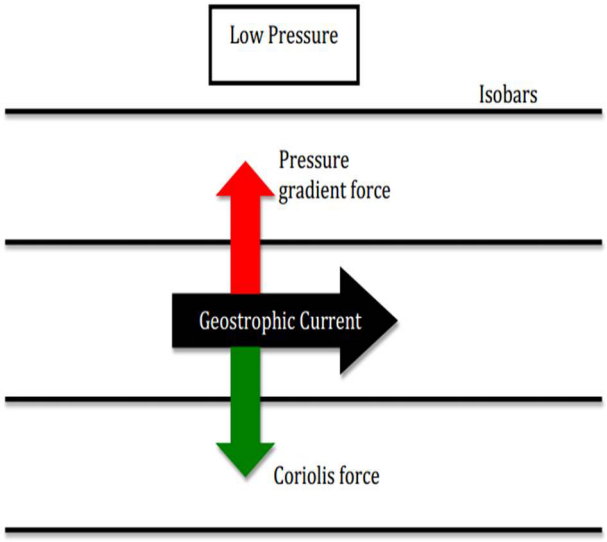
•Surface Ocean Currents:

- *in situ* measurements (drifter, buoy, etc.)
- Circulation modeling
- Geostrophy calculation from Space Geodesy

Surface geostrophic current (Pressure gradient force = Coriolis force):

$$u_s = -\frac{g}{f}\frac{\partial DH}{\partial y},$$

$$v_s = \frac{g}{f}\frac{\partial DH}{\partial x},$$

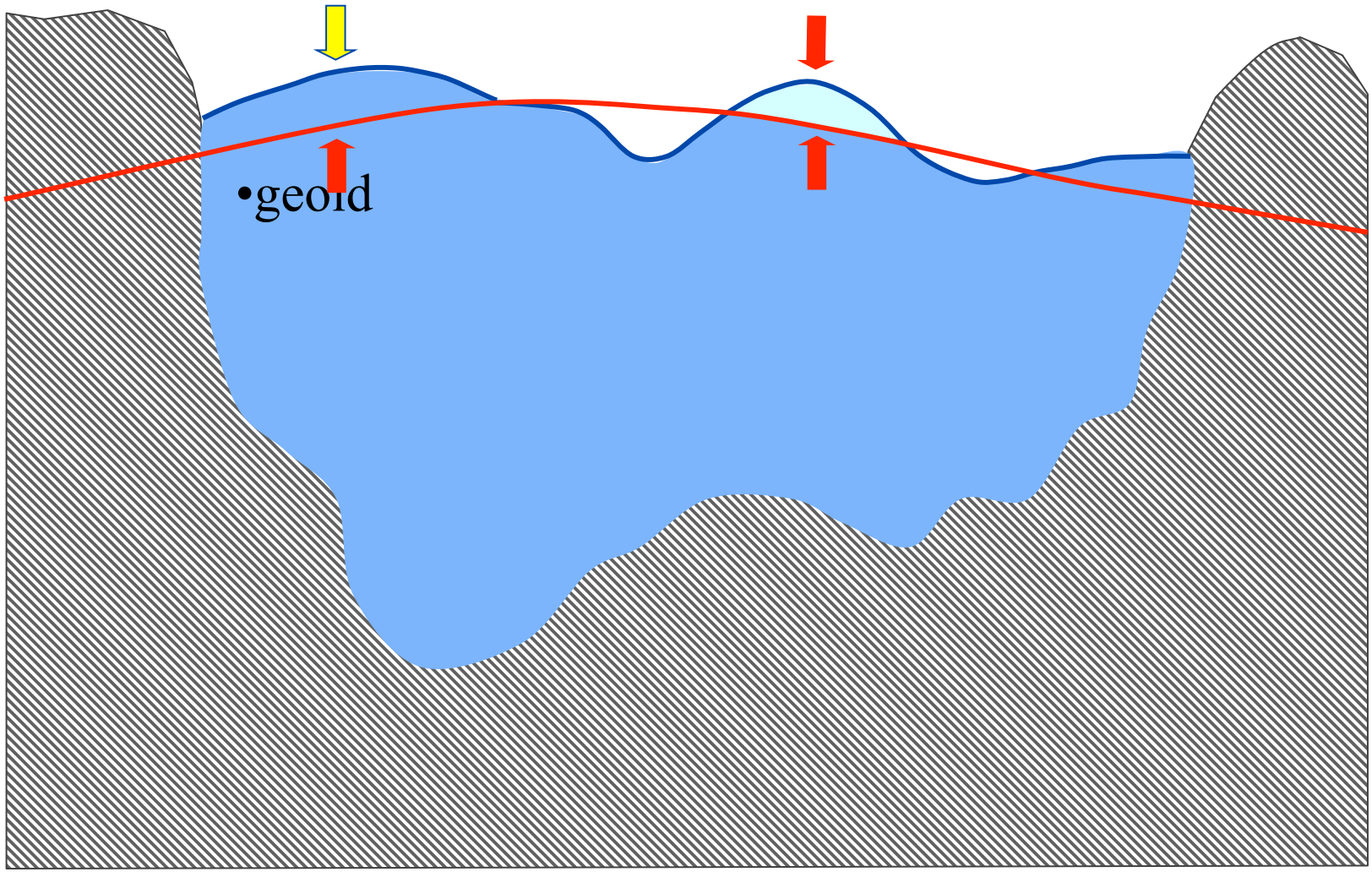


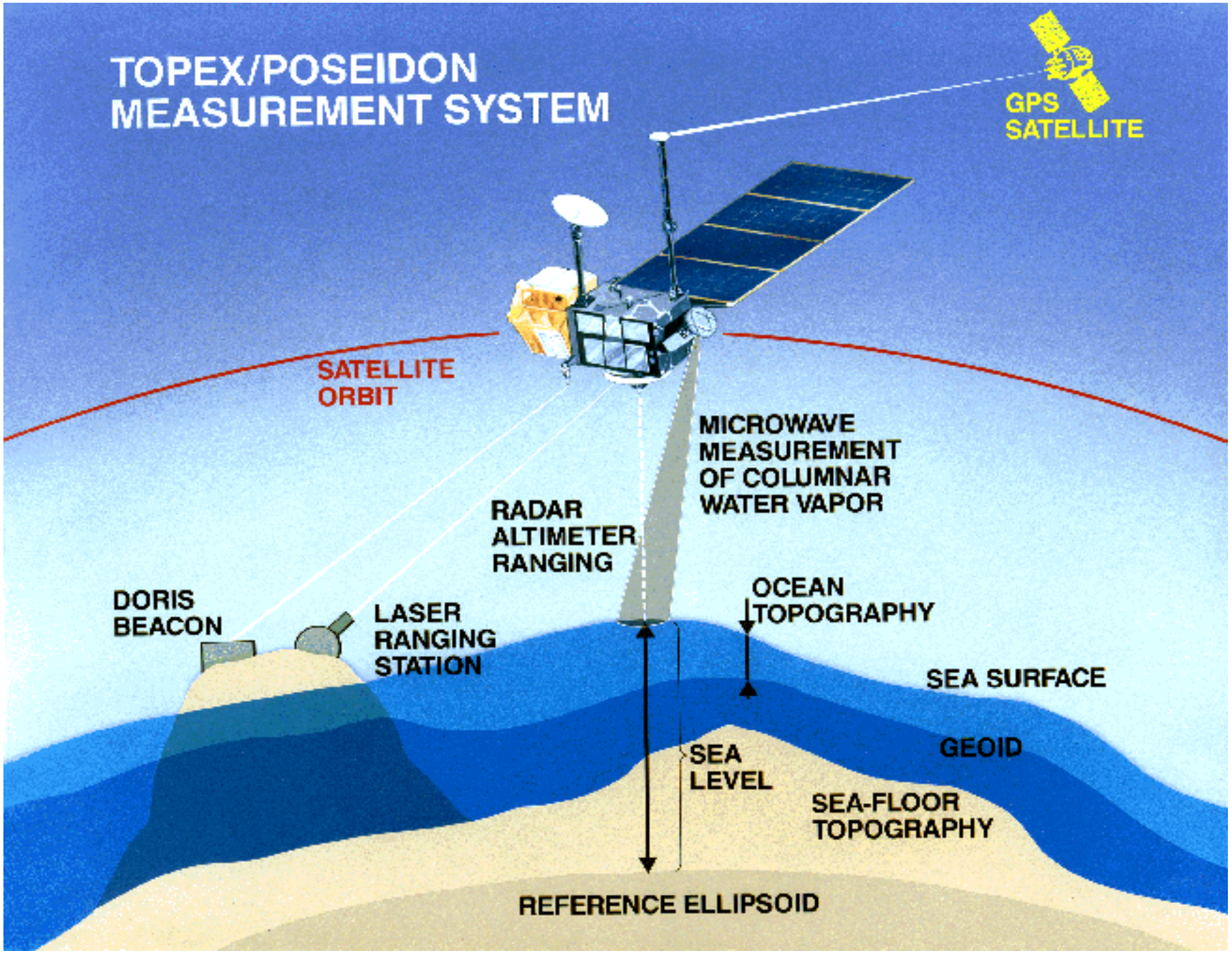
•(Northern Hemisphere)

- (u_s, v_s) = (East, North) surface current speed;
- Coriolis parameter f = 2*(Earth rotation rate)*sin(Latitude);
- ***DH* = dynamic height = true height – geoid (大地水準面).**

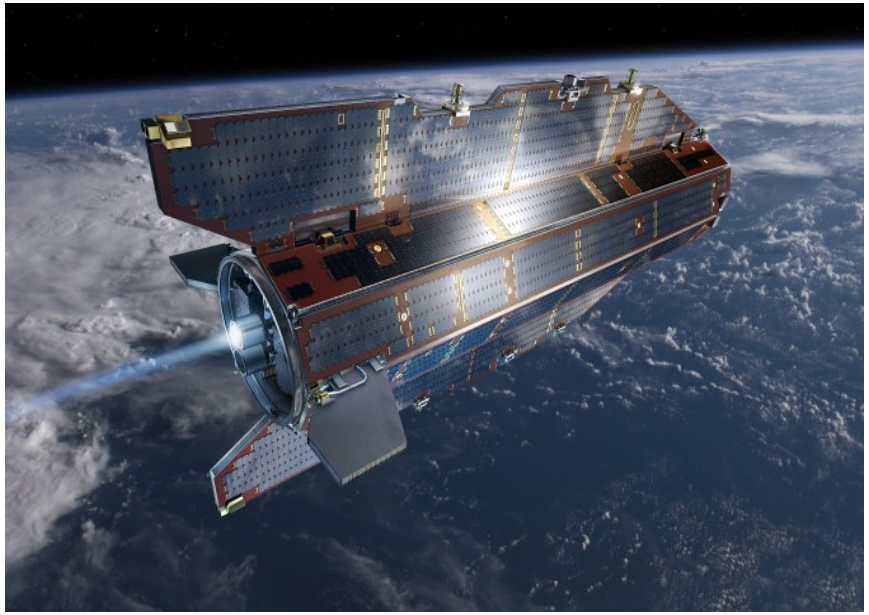
•altimetry

•dynamic ocean topography




TOPEX/POSEIDON
MEASUREMENT SYSTEM
GPS
SATELLITE
SATELLITE
ORBIT
MICROWAVE
MEASUREMENT
OF COLUMNAR
WATER VAPOR
RADAR
ALTIMETER
RANGING
DORIS
BEACON
LASER
RANGING
STATION
OCEAN
TOPOGRAPHY
SEA SURFACE
GEOID
SEA
LEVEL
SEA-FLOOR
TOPOGRAPHY
REFERENCE ELLIPSOID

GOCE: Gravity and steady-state Ocean Circulation Explorer



March 17, 2009 – November 11, 2013

• first mission of ESA's "Living Planet programme" (followed by SMOS and CRYOSAT)

• mission goals:

gravity with 1 ppm (1 mGal) accuracy

geoid with 1 to 2 cm accuracy

spatial resolution 100km

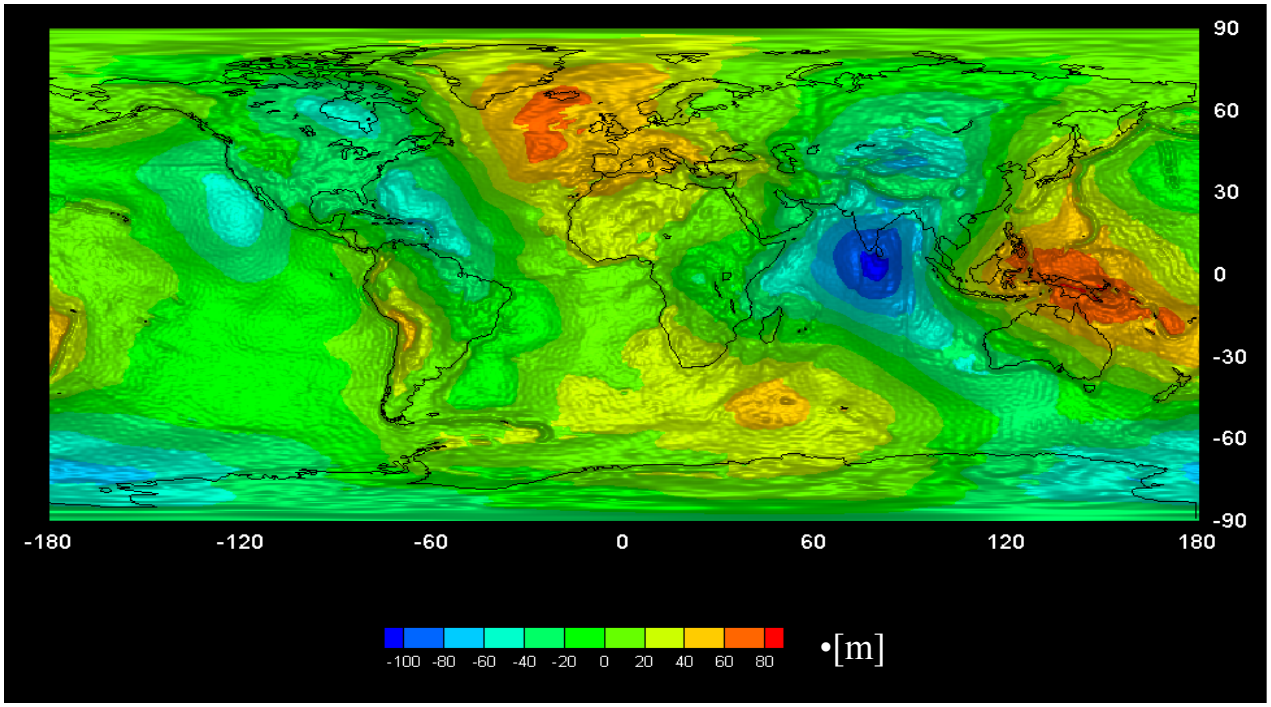
(equivalent to degree/order = 200 in spherical harmonics)

• orbit characteristics

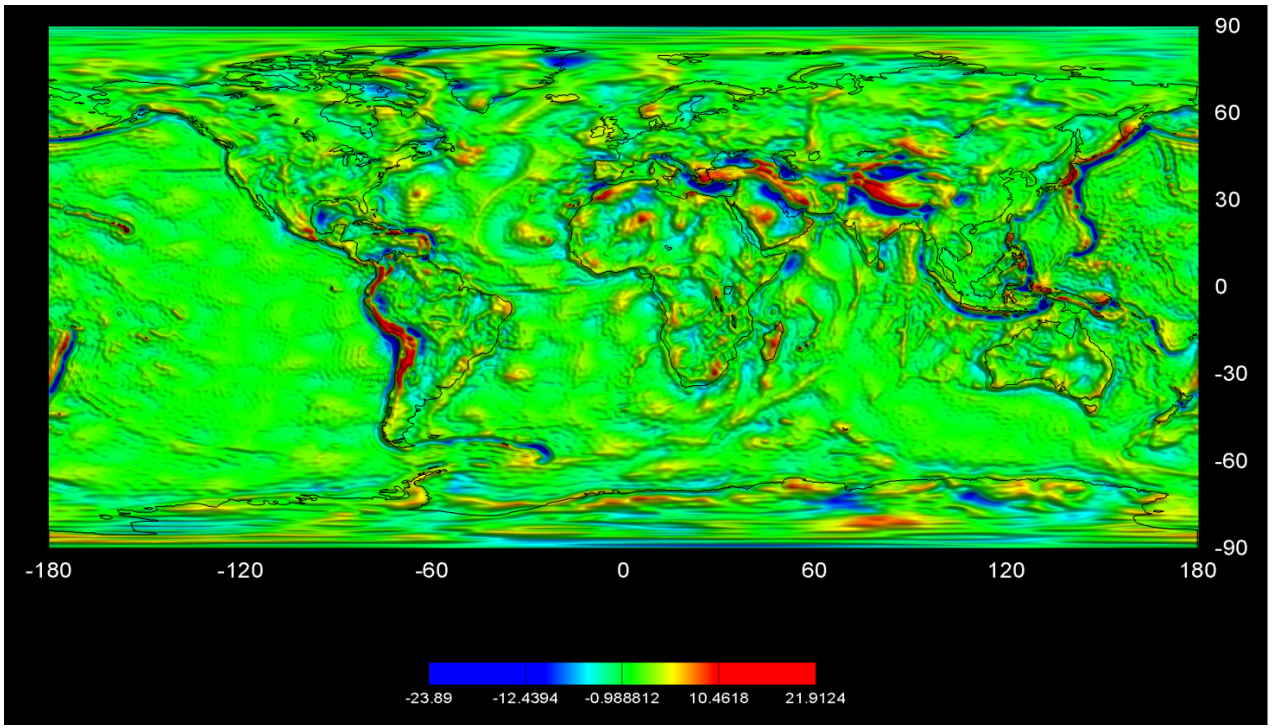
inclination 96.5° (sun-synchronous)

circular

altitude 265km (low !)



Earth geoid from GOCE



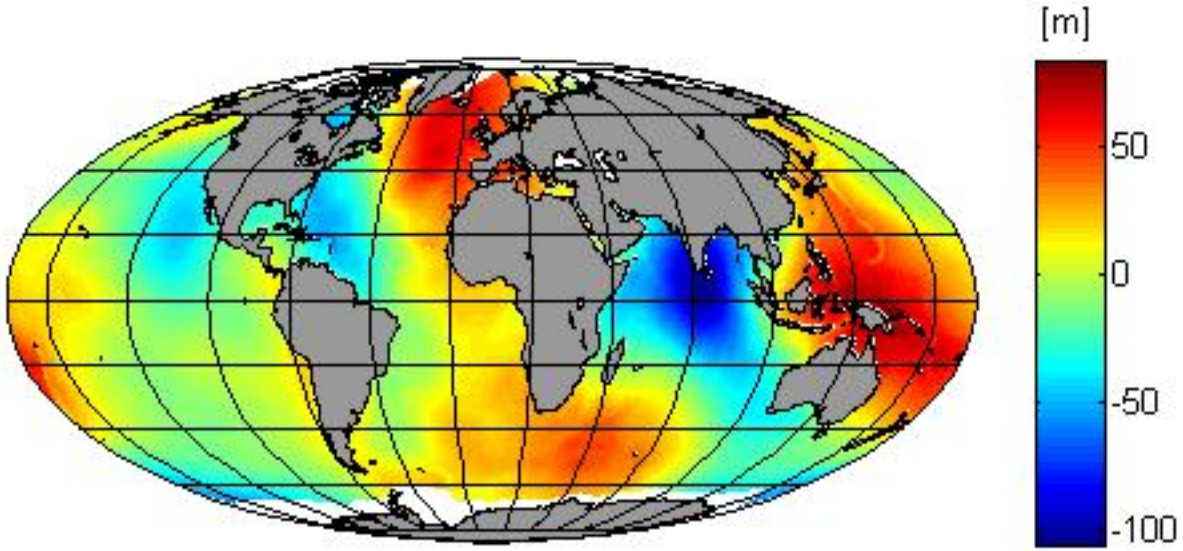
(Short-wavelength Only)

(R. Rummel)

mean ocean surface

from radar altimetry

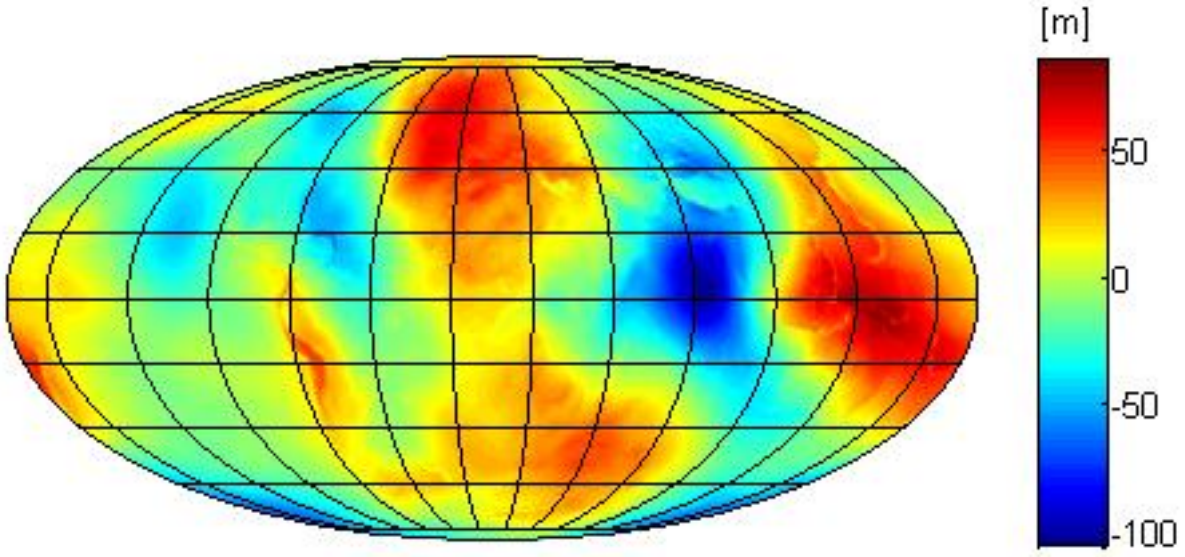
1992- 2010



geoid

from six months

GOCE data

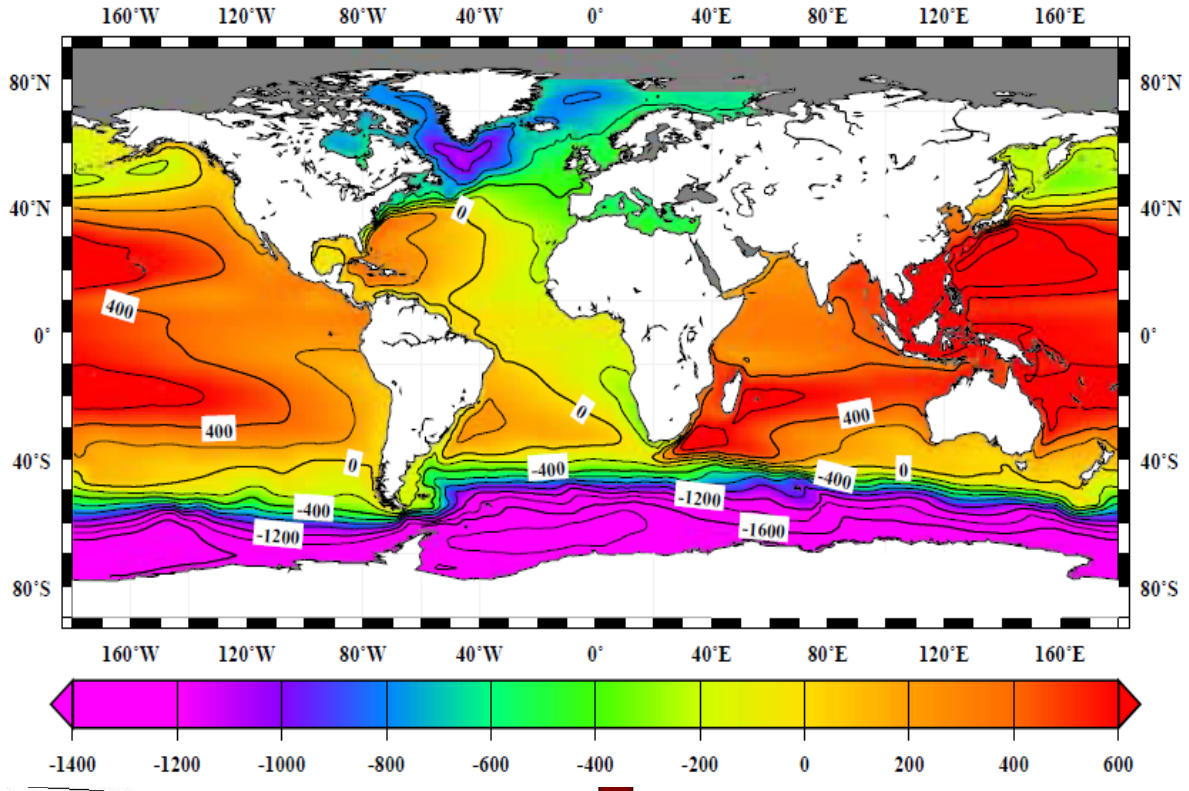


(R. Rummel)

•Mean Dynamic Topography

•= “true” mean seal level

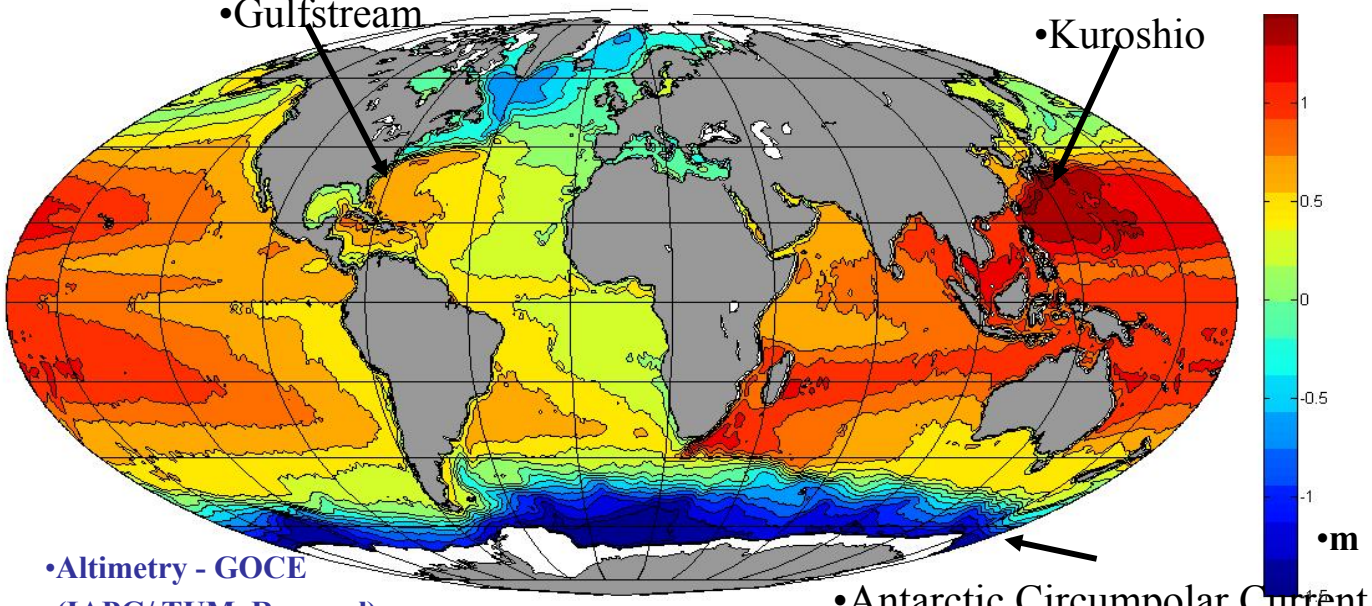
•—— Geoid



•Liverpool-MIT ocean model

•(1993-2002)

•(Woodworth et al.)



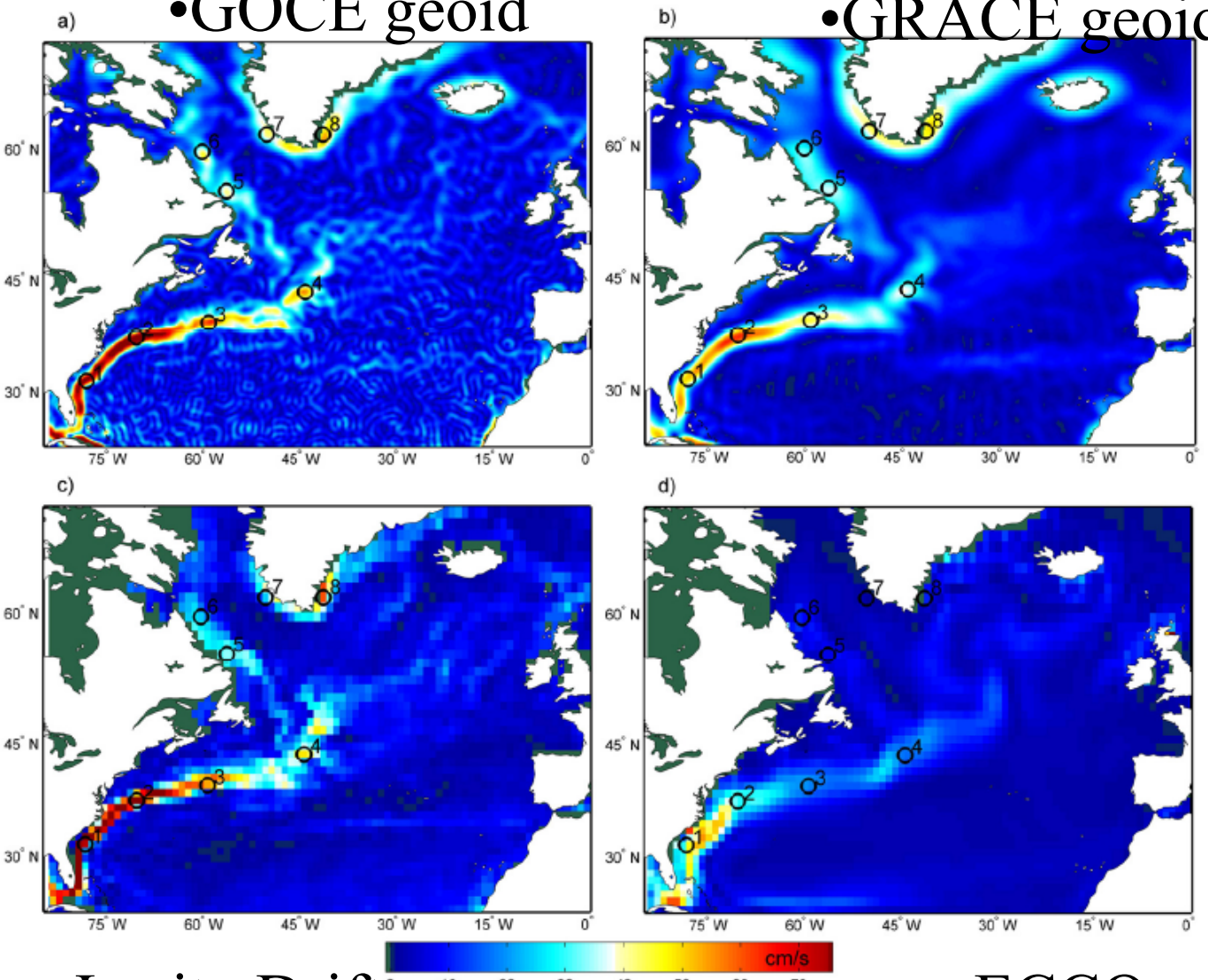
•Altimetry - GOCE

•(IAPG/ TUM, Rummel)

•Gulf Stream

•GOCE geoid

•GRACE geoid



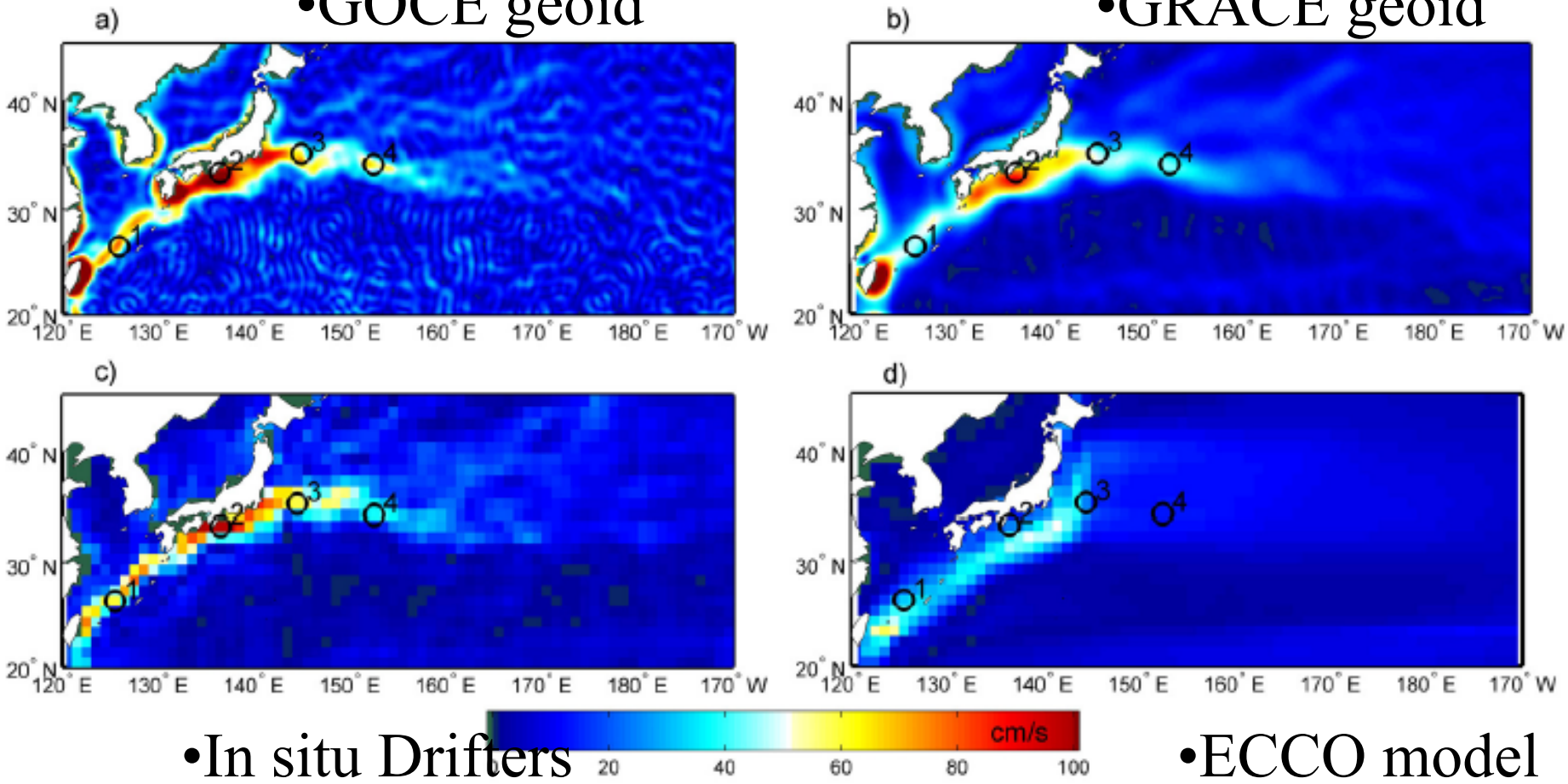
•In situ Drifters

•ECCO model

•Kuroshio

•GOCE geoid

•GRACE geoid



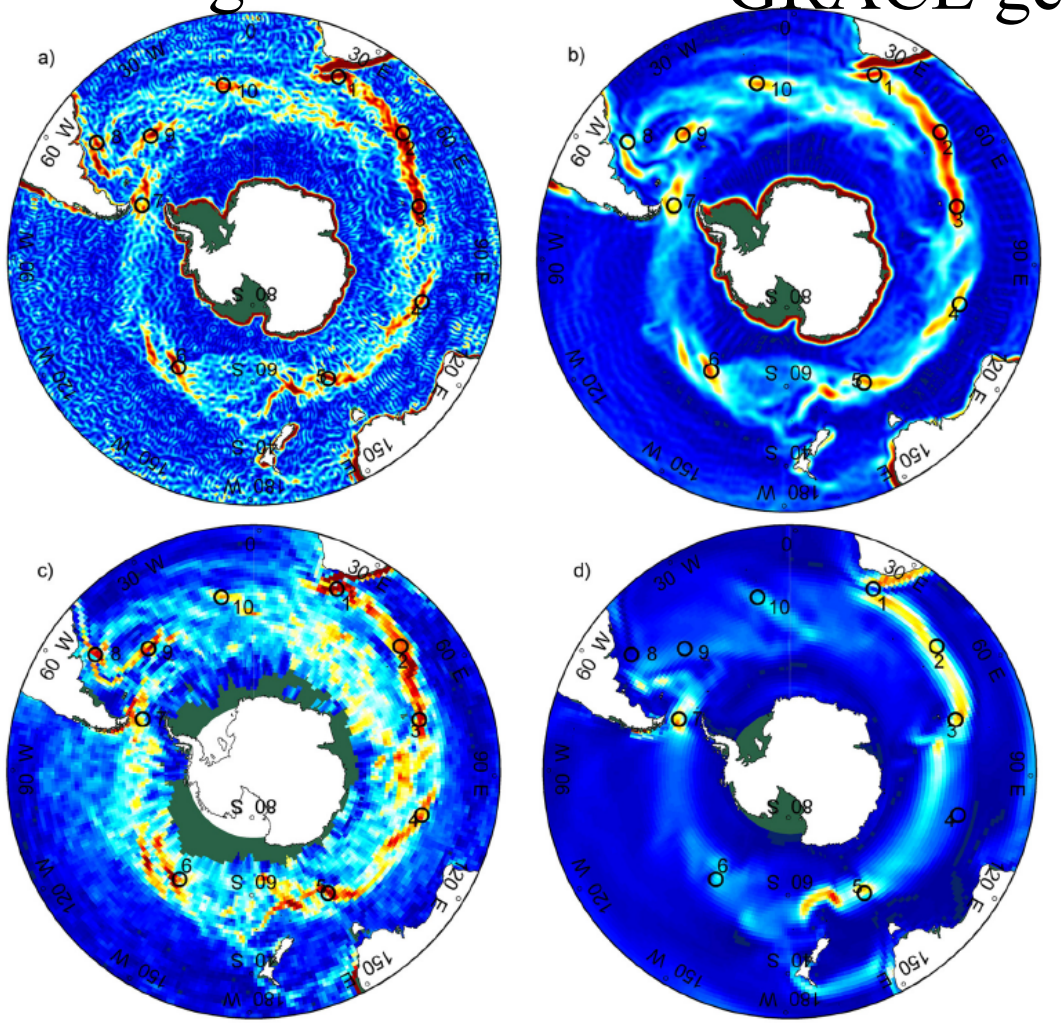
•In situ Drifters

•ECCO model

•Antarctica Circumpolar Current

•GOCE geoid

•GRACE geoid



•In situ Drifters

•ECCO model