

The Role of Satellite Geodesy in the Studies of Earth System Dynamics

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Objectives for Presentation

Consider the role of Geodetic Measurements in studying
Earth System Dynamics

Summarize recent trends in applying Space Geodesy techniques

Focus on the role of the satellite
Emphasize the role of Orbit Determination Methodology

Describe some of activities that have influenced
the development of study

Look at some recent accomplishments in gravity modeling field

Will not attempt a complete coverage of the history
or a complete attribution to individual contributions

Satellite Geodesy

Satellite geodesy is based on the use artificial satellites to determine:
the form and dimensions of the Earth,
the location of points on its surface and
the nature of the gravity field associated with it's mass

Current studies expand the definition to include repeated measurements to determine the **temporal change** in these properties (e.g. Earth rotation, crustal deformation, temporal gravity change, tides, ..)

Satellite geodesy belongs to the broader field of **Space geodesy**, which includes the surface based geodetic measurements of Very Long Baseline Interferometry(VLBI), Lunar Laser Ranging (LLR) and Surface based GPS and Doris measurements

Emphasis will be given to the role of the **satellite**

Historical Development of Satellite Geodesy

Eratosthenes (3rd Century BC) determined the Earth's circumference as $\sim 39,500$ km

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Laplace(1802) used lunar nodal motion to determine the flattening of Earth ($f = 1/303$)

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Era associated with the artificial satellite began with the launch of Sputnik in 1957

Events After 1957

1957 Launch of SPUTNIK-1

Vanguard TV3 explodes shortly after launch

1958 Launch of EXPLORER-1

Launch of Vanguard

NASA was formed

Earth's Flattening from Satellite Data ($f=1/298.3$)

1959 Third Zonal Harmonic (Pear Shape of Earth),

Theory of the Motion of Artificial Satellites (Brouwer),

1960 Launch of TRANSIT-1B, (Operational Navigation System in 1964)

Launch of ECHO-1

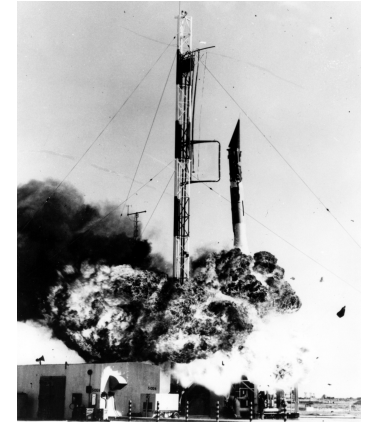
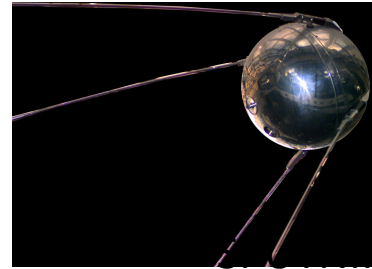
Theory of Satellite Orbits (Kaula),

1962 Launch of ANNA-1B

Geodetic Connection between France and Algeria (IGN).

By 1964, many basic geodetic problems had been successfully tackled,

- determination of the Earth's flattening($f=1/298.3$)
- determination of the Third Zonal Harmonic(Pear Shape of the Earth)
- determination of the general shape of the global geoid
- connections between the most important geodetic datums (to ± 50 m)



TV3 Launch



Explorer-1 Launch

Williamstown Conference 1969

Objective:

to explore the possible contributions of accurate position, velocity, and acceleration measurements to the solution of problems in solid- earth and ocean physics and to make program recommendationsregarding understanding of the ocean circulation and of the crust and mantle tectonics .

Implementation involved a

Central Review Committee (Chaired by Bill Kaula)

and four panels:

Instrumentation (Chaired by George C. Weiffenbach)

Ocean Physics (Chaired by William S. von Arx;

Short-term Dynamics of the Solid Earth (Chaired by Charles A. Lundquist

Long – term Dynamics of the Solid Earth (Chaired by Lynn R . Sykes)

Participants involved over 65 scientists, including 15 NASA officials from the Solid Earth, Oceanographic, Instrumentation and Satellite Data Analysis fields

Measurement Objectives from the WTC

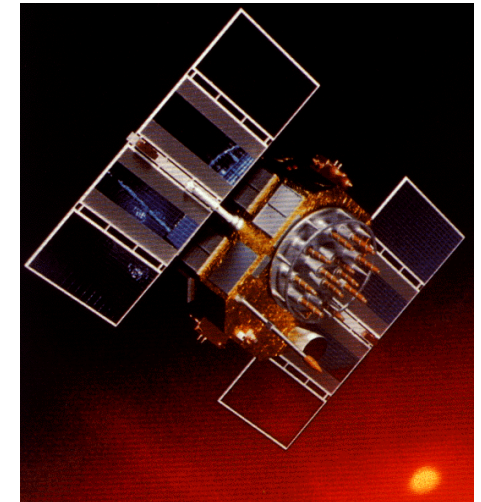
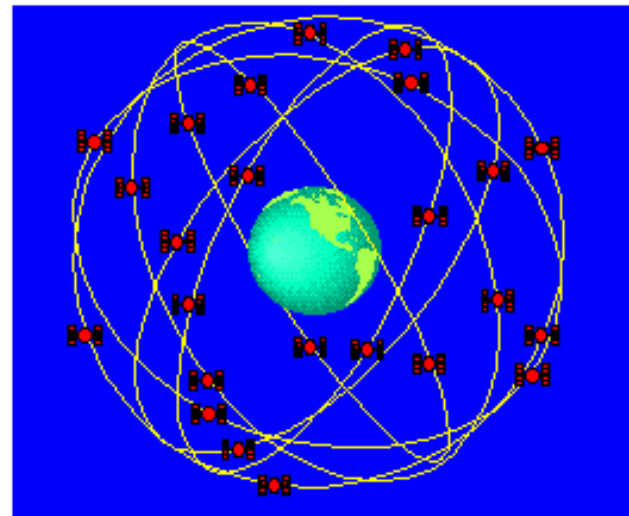
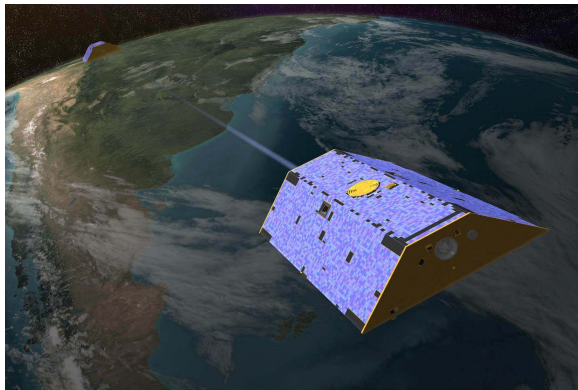
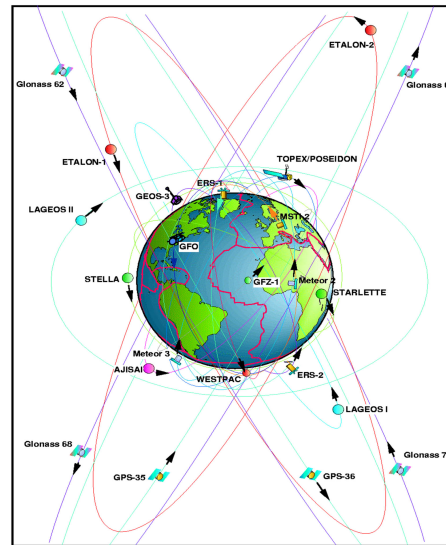
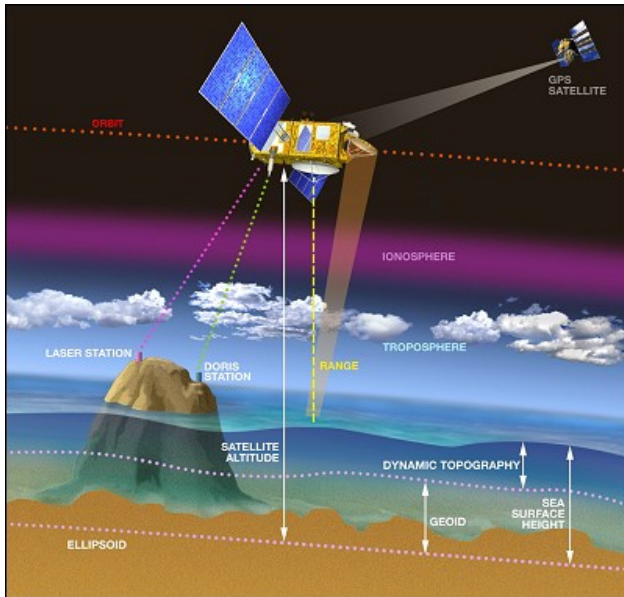
- Spatial variations of the geoid to ± 10 cm.
- Measurement of mean sea level on a global basis to ± 10 cm over a continuous spectrum from periods of 10 sec to many years.
- Measurements of time variations in the geoid to ± 5 cm for well-defined tidal frequencies plus random, seasonal, and secular variations.
- Variations in rate of rotation of the earth, and wandering of the pole to accuracies of ± 0.005 for averaging times of 1 day.
- Relative movement of sections of the earth's surface to an accuracy of ± 10 cm or better in an inertial reference system.
- Relative motions of earth's surface across faults.

Recommendations from Williamstown Report

- Altimeter Mission
 - Drag Free, 10 cm range accuracy, high low tracking from 6000 km
 - Determine the general Ocean Circulation
 - Determine the Marine geoid at 100 km half wave lengths
- Gravity Mission
 - Dual satellite High/Low Mission
 - Range and range- rate tracking: ± 10 - cm and ± 0.05 -mm /sec accuracy, 5-sec smoothing time; circular or bit, preferably about 2 00-lan altitude, predominantly polar inclination for the close satellite, occasional intermediate inclination.
 - Global Mean Gravity Model with 100 km spatial resolution
- Laser Ranging and VLBI:
 - ± 2 -cm accuracy.
 - For generation of a time history of tectonic plate motion
- Economical ground beacons or transponders:
 - Accuracy ± 1 m.
 - Untended operation for a wide variety of uses such as geophysical surveys....

Importance of Orbit Accuracy and need for improvement was noted

POD and Space Geodesy Applications



1-08-2008

SEASAT

Seasat was the first Earth-Orbiting Satellite designed for remote sensing of the Earth's Oceans.

An Oceanographic satellite that had an enormous geodetic impact

Launched on 27 June 1978 into a nearly circular 800 km orbit with an inclination of 108°.

Seasat operated for 106 days until 10 October 1978, when the mission ended due to a failure in the electrical power system

Seasat carried five major instruments:

- Radar altimeter to measure spacecraft height above the ocean surface
- Microwave Scatterometer to measure wind speed and direction
- Scanning multichannel microwave radiometer to measure sea surface temperature
- Visible and infrared radiometer to identify cloud, land and water features
- Synthetic Aperture Radar (SAR) L-band, to monitor the global surface wave field and polar sea ice conditions



SEASAT Space Geodesy Impact

SAR

ERS-1, ERS-2 and Envisat, ESA
Radar Sat-1, Radar Sat-2, CSA
TerraSAR-X and TanDEM-X , Germany
SRTM, NGA/NASA
NASA-ISRO (NI-SAR)

Satellite Altimetry

Topex and Jason Satellite Series, NASA/ CNES
GeoSat and Geosat FO, NAVY/NOAA
ERS-1,2 and EnviSat ESA
HY-2 Series, China

Ancillary

Calibrated over-flight calibration concept used by
future missions
Proposed GPS Receiver on SeaSat would have been
first Satellite based GPS Receiver.
Stimulated SERIES X receiver development at JPL

Contributions to Satellite Geodesy

- MOBLIS SLR Tracking Systems were developed and deployed to track Seasat
- Stimulated development of first satellite born GPS receiver
- Recognition of need for Precise Orbit Determination

The Altimeter Mission

Global MSL, Ocean Circulation, Marine Geoid
and Ocean Bathymetry

The Satellite Altimeter Missions

Wide Swath Altimeter(SWOT) NASA 2020

Jason-3 CNES/EUMETSAT/ NASA/NOAA 2015

Sentinel-3 ESA 2013*

Saral CNES/ISRO 2010*

Cryosat ESA 2009*

HY-2 CNSA 2009, 2012*, 2015

Envisat ESA 2002

Jason-2 CNES/EUMETSAT/ NASA/NOAA 2008*

Jason-1 CNES/NASA 2001

GeoSat FO US Navy/NOAA 1998

ERS-2 ESA 1995

ERS-1 ESA 1991

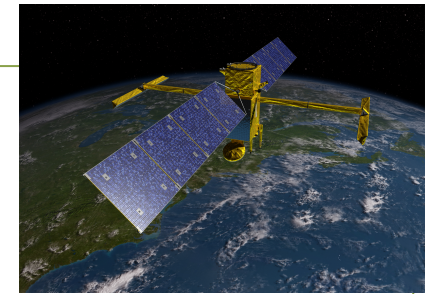
Topex/Poseidon NASA/CNES 1992

GeoSat US Navy 1985

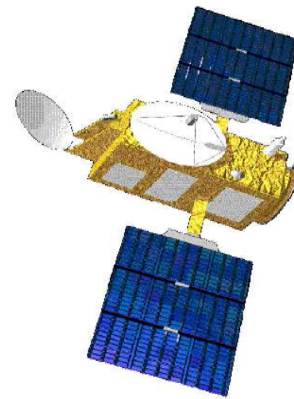
SeaSat NASA/US NAVY 1978

GEOS 3 US NAVY 1976

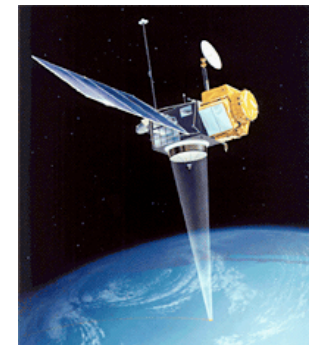
SkyLab NASA 1973



SWOT



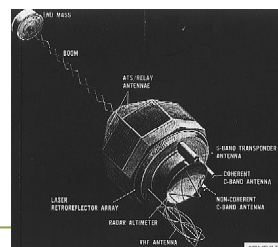
Jason 1



T/P

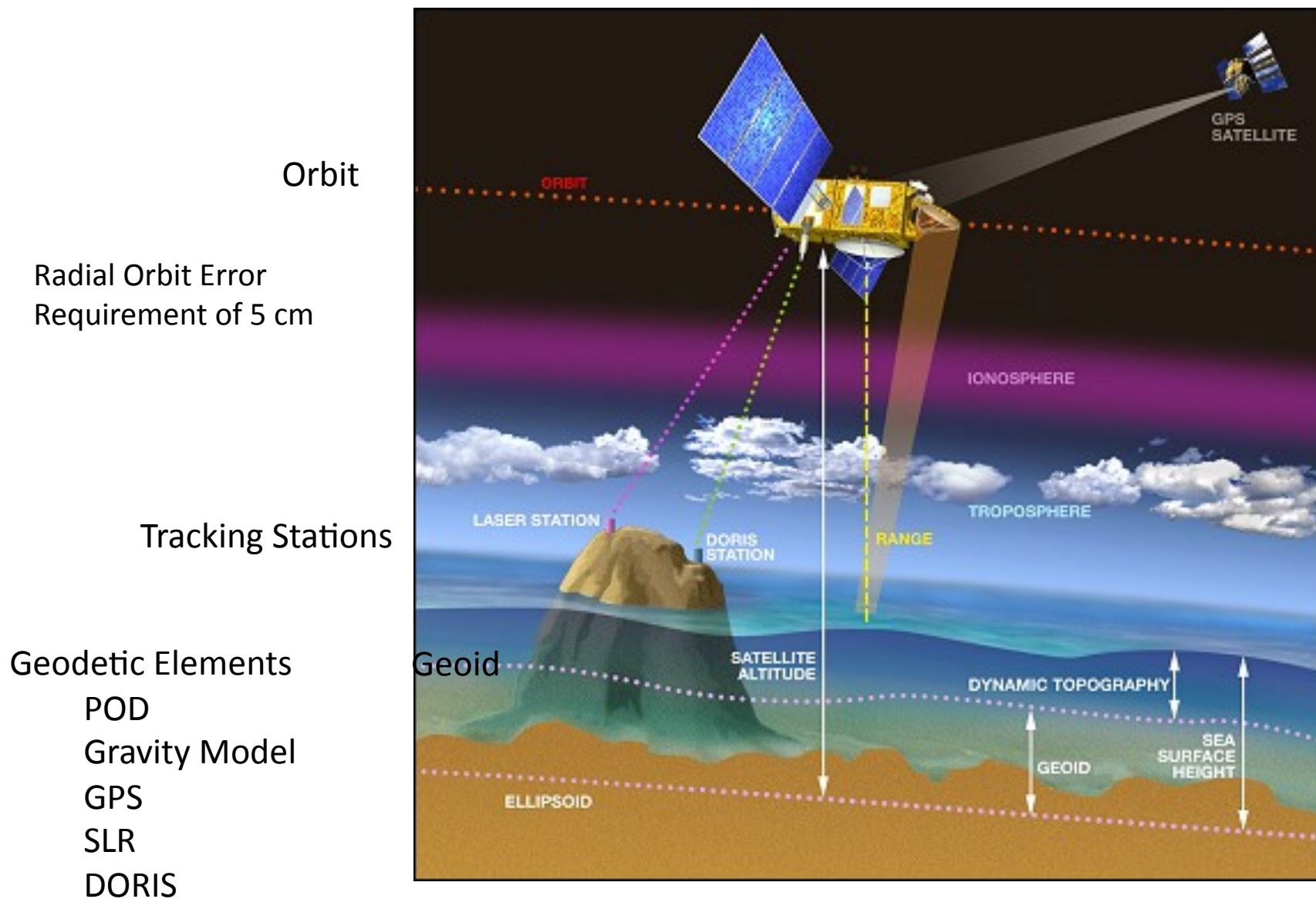


SeaSat



GEOS 3

TOPEX/POSEIDON Ocean Topography Mission



TOPEX/POSEIDON Measurement Accuracy

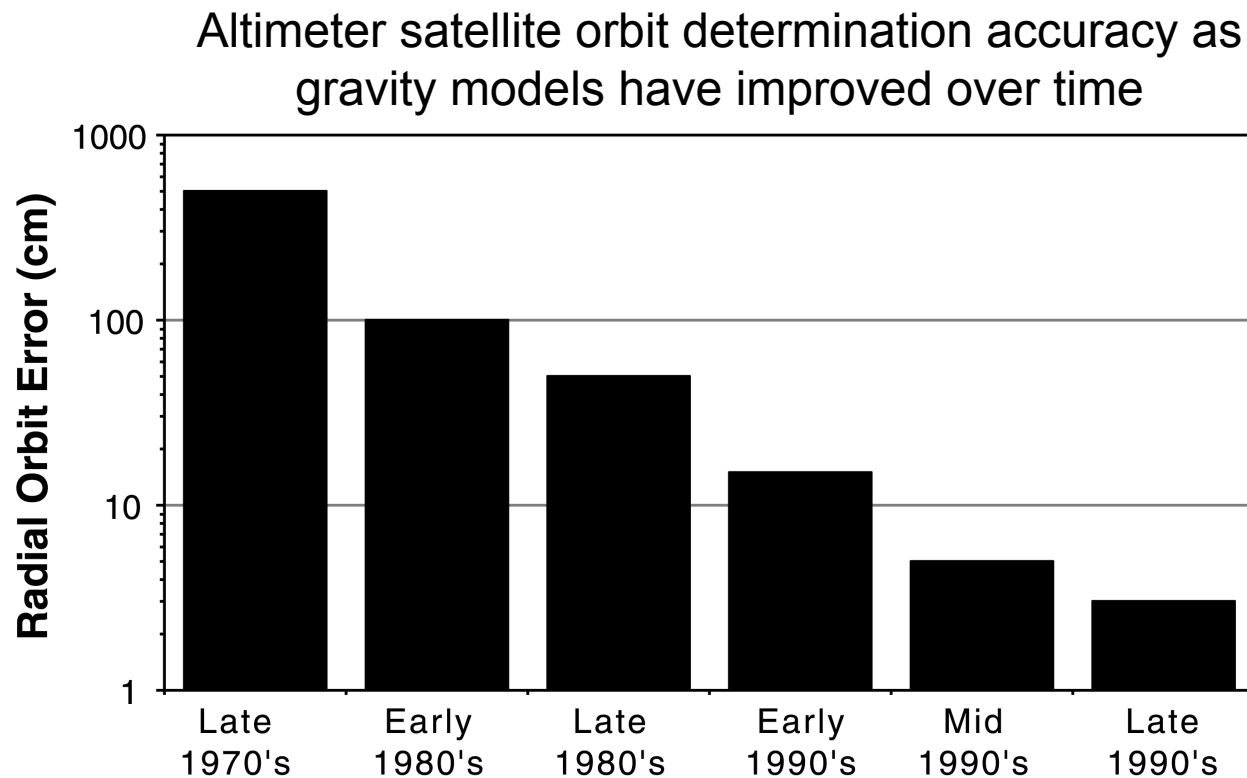
Tracking systems used for POD (Precision Orbit Determination)

- 1) SLR (Satellite Laser Ranging) to the LRA (Laser Ranging Array)
- 2) DORIS
- 3) GPSDR (An experimental satellite GPS receiver)

Measurement	Required accuracy	Achieved accuracy
Altimeter	4.0 cm	3.2 cm
Satellite position	12.8 cm	2.3 cm
SSH (Sea Surface Height)	13.4 cm	3.9 cm

Gravitational Force Models Effects of Altimeter Data

- Central body (point-mass or two-body)
- Non-spherical Earth perturbations from spherical harmonics up to degree and order 99



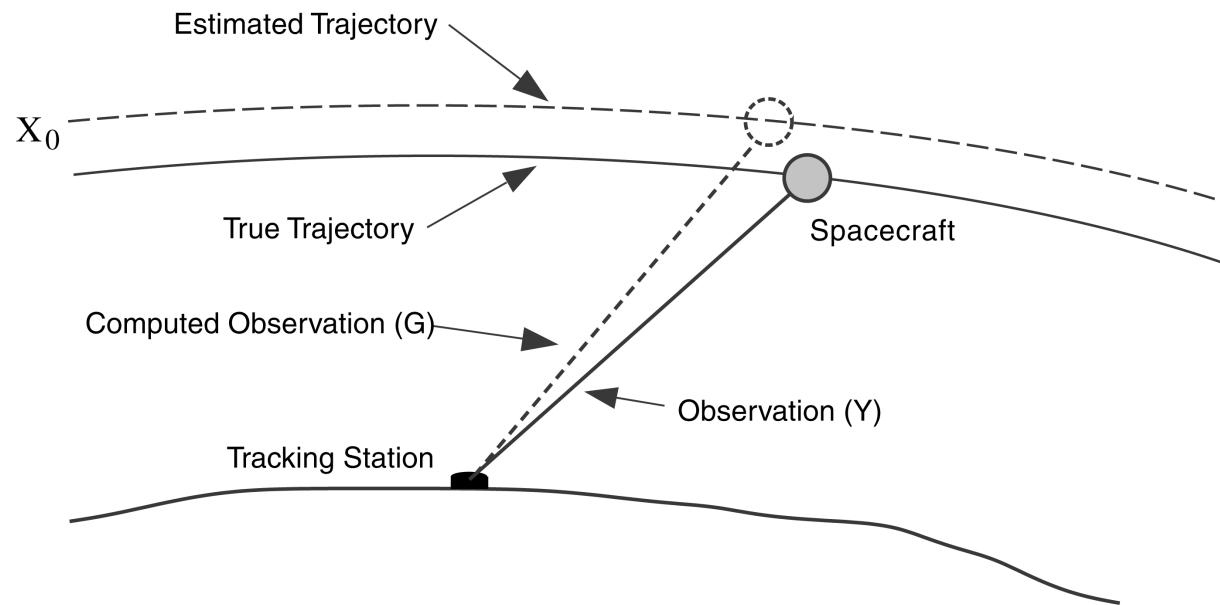
Gravity Model Impact of POD Accuracy

Laser range and altimeter crossover residual RMS using various gravity models for TOPEX/POSEIDON and ERS-2. The larger errors for ERS-2 reflect the increased gravity errors and higher atmospheric drag associated with the lower altitude, as well as a lower level of tracking coverage compared to TOPEX/POSEIDON.

Gravity Model	Crossover RMS (cm)		Laser Range RMS (cm)	
	T/P	ERS-2	T/P	ERS-2
GEM-10B (<i>Lerch et al.</i> , 1981)	66	267	45	55
GEM-T1 (<i>Marsh et al.</i> , 1989)	40	205	20	56
GEM-T2 (<i>Marsh et al.</i> , 1990)	18	148	10	58
TEG-2B (<i>Shum et al.</i> , 1990a)	12	12	7	12
GEM-T3 (<i>Lerch et al.</i> , 1993)	9	58	6	48
JGM-1 (<i>Nerem et al.</i> , 1994)	7	9.9	4	14
JGM-2 (<i>Nerem et al.</i> , 1994)	6.5	9.6	2.7	13
GRIM4-C4 (<i>Schwintzer et al.</i> , 1997)	6.5	9.2	2.6	6.9
JGM-3 (<i>Tapley et al.</i> , 1996)	6.1	8.4	2.3	6.8
EGM96 (<i>Lemoine et al.</i> , 1998)	6.2	8.0	2.4	6.8
TEG-3 (<i>Tapley et al.</i> , 1997)	6.1	7.5	2.3	3.3
DGM-E04 (<i>Scharroo and Visser</i> , 1998)	6.1	6.7	2.3	4.3
JGM-3P/TEG-3P (<i>Bordi et al.</i> , 2000)	6.1	6.7	2.3	3.1

Fundamentals of Orbit Determination

The Orbit Determination Problem



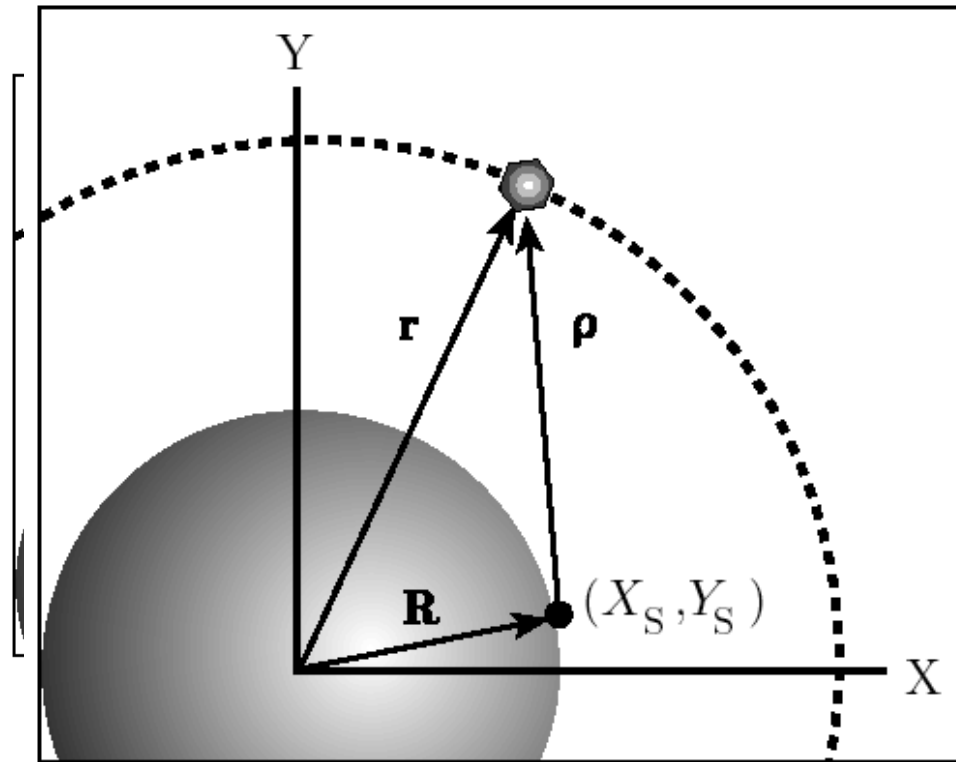
$$\dot{X}^* = F(X^*, t), \quad X^*_{(t_0)} = X_0^*$$

$$Y = G(X^*, t) + \varepsilon^*$$

Example 4.2.1

Compute the A matrix and the $\tilde{H}$ matrix for a satellite in a plane under the influence of only a *central force*. Assume that the satellite is being tracked with range observations, ρ , from a single ground station. Assume that the station coordinates, (X_S, Y_S) , and the gravitational parameter are unknown. Then, the state vector, $\mathbf{X}$, is given by

$$\mathbf{X} = \begin{bmatrix} X \\ Y \\ U \\ V \\ \mu \\ X_S \\ Y_S \end{bmatrix}$$



where U and V are velocity components and X_S and Y_S are coordinates of the tracking station. From Newton's Second Law and the law of gravitation,

$$\ddot{\mathbf{r}} = -\frac{\mu \mathbf{r}}{r^3} = \mathbf{f}(\mathbf{r}, \mathbf{v}, \mathbf{c}, \mathbf{t})$$

Then, the nonlinear orbit determination problem in which the complete state vector is to be estimated can be replaced by a linear orbit determination problem in which the deviation from some reference solution is to be determined.

To conduct this linearization procedure, let the $n \times 1$ state deviation vector, $\mathbf{x}$, and the $p \times 1$ observation deviation vector, $\mathbf{y}$, be defined as follows:

$$\mathbf{x}(t) = \mathbf{X}(t) - \mathbf{X}^*(t), \quad \mathbf{y}(t) = \mathbf{Y}(t) - \mathbf{Y}^*(t) \quad (4.2.3)$$

It follows that

$$\dot{\mathbf{x}}(t) = \dot{\mathbf{X}}(t) - \dot{\mathbf{X}}^*(t). \quad (4.2.4)$$

Expanding Eqs. (4.2.1) and (4.2.2) in a Taylor's series about the reference trajectory leads to

$$\begin{aligned} \dot{\mathbf{X}}(t) &= F(\mathbf{X}, t) = F(\mathbf{X}^*, t) + \left[\frac{\partial F(t)}{\partial \mathbf{X}(t)} \right]^* [\mathbf{X}(t) - \mathbf{X}^*(t)] \\ &\quad + O_F [\mathbf{X}(t) - \mathbf{X}^*(t)] \\ \mathbf{Y}_i &= G(\mathbf{X}_i, t_i) + \boldsymbol{\epsilon}_i = G(\mathbf{X}_i^*, t_i) + \left[\frac{\partial G}{\partial \mathbf{X}} \right]_i^* [\mathbf{X}(t_i) - \mathbf{X}^*(t_i)] \\ &\quad + O_G [\mathbf{X}(t_i) - \mathbf{X}^*(t_i)] + \boldsymbol{\epsilon}_i \end{aligned} \quad (4.2.5)$$

Look at first equation in (4.2.5).

$$\dot{X}(t) = F(X^*, t) + \left[\frac{\partial F(t)}{\partial X(t)} \right]^* [X(t) - X^*(t)] + H.O.T.$$

Drop H.O.T. Using the following,

$$\dot{X}^*(t) = F(X^*, t)$$

$$x(t) = X(t) - X^*(t)$$

$$A(t) \equiv \left[\frac{\partial F(t)}{\partial X(t)} \right]^*$$

gives

$$\dot{X}(t) = \dot{X}^*(t) + A(t)x(t)$$

$$\dot{X}(t) - \dot{X}^*(t) = A(t)x(t)$$

$$\boxed{\dot{x}(t) = A(t)x(t)}$$

or in component form,

$$\ddot{X} = -\frac{\mu X}{r^3}$$

$$\ddot{Y} = -\frac{\mu Y}{r^3}.$$

If these equations are expressed in first-order form, the following expression is obtained:

$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{U} \\ \dot{V} \\ \dot{\mu} \\ \dot{X}_S \\ \dot{Y}_S \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \end{bmatrix} = \begin{bmatrix} U \\ V \\ -\frac{\mu X}{r^3} \\ -\frac{\mu Y}{r^3} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \mathbf{F}$$

$$A(t) = \frac{\partial F(\mathbf{X}^*, t)}{\partial \mathbf{X}} = \begin{bmatrix} \frac{\partial F_1}{\partial X} & \frac{\partial F_1}{\partial Y} & \frac{\partial F_1}{\partial U} & \frac{\partial F_1}{\partial V} & \frac{\partial F_1}{\partial \mu} & \frac{\partial F_1}{\partial X_S} & \frac{\partial F_1}{\partial Y_S} \\ \frac{\partial F_2}{\partial X} & \dots & \dots & \dots & \dots & \dots & \frac{\partial F_2}{\partial Y_S} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial F_7}{\partial X} & \dots & \dots & \dots & \dots & \dots & \frac{\partial F_7}{\partial Y_S} \end{bmatrix}^*$$

$$A(t) = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -\frac{\mu}{r^3} + \frac{3\mu X^2}{r^5} & \frac{3\mu XY}{r^5} & 0 & 0 & -\frac{X}{r^3} & 0 & 0 \\ \frac{3\mu XY}{r^5} & -\frac{\mu}{r^3} + \frac{3\mu Y^2}{r^5} & 0 & 0 & -\frac{Y}{r^3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^*$$

Look at second equation in (4.2.5).

$$Y_i = G(X_i^*, t_i) + \left[\frac{\partial G}{\partial X} \right]_i^* [X(t_i) - X^*(t_i)] + H.O.T. + \varepsilon_i$$

Drop H.O.T. Using the following,

$$Y_i^* \equiv G(X_i^*, t_i)$$

$$\tilde{H}_i \equiv \left[\frac{\partial G}{\partial X} \right]_i^*$$

$$x(t) = X(t) - X^*(t)$$

$$y_i \equiv Y_i - Y_i^* = Y_i - G(X_i^*, t_i)$$

gives

$$Y_i = Y_i^* + \tilde{H}_i x_i + \varepsilon_i$$

$$Y_i - Y_i^* = \tilde{H}_i x_i + \varepsilon_i$$

$$\boxed{y_i = \tilde{H}_i x_i + \varepsilon_i}$$

The $\tilde{H}$ matrix is given by

$$\tilde{H} = \frac{\partial \rho}{\partial \mathbf{X}} = \left[\begin{array}{ccccccc} \frac{\partial \rho}{\partial X} & \frac{\partial \rho}{\partial Y} & \frac{\partial \rho}{\partial U} & \frac{\partial \rho}{\partial V} & \frac{\partial \rho}{\partial \mu} & \frac{\partial \rho}{\partial X_S} & \frac{\partial \rho}{\partial Y_S} \end{array} \right]^*$$

where

$$\rho = \left[(X - X_S)^2 + (Y - Y_S)^2 \right]^{1/2}.$$

It follows then that

$$\tilde{H} = \left[\begin{array}{ccccccc} \frac{X - X_S}{\rho} & \frac{Y - Y_S}{\rho} & 0 & 0 & 0 & -\frac{(X - X_S)}{\rho} & -\frac{(Y - Y_S)}{\rho} \end{array} \right]^*$$

If the terms of order higher than the first in Eq. (4.2.5) are neglected, under the assumption that the higher order products are small compared to the first order terms, and if the condition $\dot{\mathbf{X}}^* = F(\mathbf{X}^*, t)$ and $\mathbf{Y}_i^* = G(\mathbf{X}_i^*, t_i)$ are used, Eq. (4.2.5) can be written as

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) \quad (4.2.6)$$

$$\mathbf{y}_i = \tilde{H}_i \mathbf{x}_i + \boldsymbol{\epsilon}_i \quad (i = 1, \dots, \ell)$$

where

$$A(t) = \left[\frac{\partial F(t)}{\partial \mathbf{X}(t)} \right]^* \quad \tilde{H}_i = \left[\frac{\partial G}{\partial \mathbf{X}} \right]_i^*.$$

Hence, the original nonlinear estimation problem is replaced by the linear estimation problem described by Eq. (4.2.6), where

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{X}(t) - \mathbf{X}^*(t), \\ \mathbf{x}_i &= \mathbf{X}(t_i) - \mathbf{X}^*(t_i) \\ \mathbf{y}_i &= \mathbf{Y}_i - G(\mathbf{X}_i^*, t_i). \end{aligned}$$

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) \quad (4.2.6)$$

$$\mathbf{y}_i = \tilde{H}_i \mathbf{x}_i + \boldsymbol{\epsilon}_i \quad (i = 1, \dots, \ell)$$

$$A(t) = \left[\frac{\partial F(t)}{\partial \mathbf{X}(t)} \right]^* \quad \tilde{H}_i = \left[\frac{\partial G}{\partial \mathbf{X}} \right]_i^*$$

4.2.1 THE STATE TRANSITION MATRIX

The first of Eq. (4.2.6) represents a system of linear differential equations with time-dependent coefficients. The symbol $[]^*$ indicates that the values of $\mathbf{X}$ are derived from a particular solution to the equations $\dot{\mathbf{X}} = F(\mathbf{X}, t)$ which is generated with the initial conditions $\mathbf{X}(t_0) = \mathbf{X}_0^*$. The general solution for this system, $\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t)$, can be expressed as

$$\mathbf{x}(t) = \Phi(t, t_k)\mathbf{x}_k \quad (4.2.7)$$

where $\mathbf{x}_k$ is the value of $\mathbf{x}$ at t_k ; that is, $\mathbf{x}_k = \mathbf{x}(t_k)$. The matrix $\Phi(t_i, t_k)$ is called the state transition matrix and was introduced in Chapter 1, Section 1.2.5.

4.2.3 RELATING THE OBSERVATIONS TO AN EPOCH STATE

$$\mathbf{x}(t) = \Phi(t, t_k)\mathbf{x}_k \quad (4.2.7)$$

$$\mathbf{y}_i = \tilde{H}_i\mathbf{x}_i + \boldsymbol{\epsilon}_i \quad (i = 1, \dots, \ell)$$

$$\begin{aligned} \mathbf{y}_1 &= \tilde{H}_1\Phi(t_1, t_k)\mathbf{x}_k + \boldsymbol{\epsilon}_1 \\ \mathbf{y}_2 &= \tilde{H}_2\Phi(t_2, t_k)\mathbf{x}_k + \boldsymbol{\epsilon}_2 \\ &\vdots \\ \mathbf{y}_\ell &= \tilde{H}_\ell\Phi(t_\ell, t_k)\mathbf{x}_k + \boldsymbol{\epsilon}_\ell. \end{aligned} \quad (4.2.37)$$

If the following definitions are used

$$\mathbf{y} \equiv \begin{bmatrix} y_1 \\ \vdots \\ y_\ell \end{bmatrix}; \quad H \equiv \begin{bmatrix} \tilde{H}_1 \Phi(t_1, t_k) \\ \vdots \\ \tilde{H}_\ell \Phi(t_\ell, t_k) \end{bmatrix}; \quad \boldsymbol{\epsilon} \equiv \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_\ell \end{bmatrix} \quad (4.2.38)$$

and if the subscript on $\mathbf{x}_k$ is dropped for convenience, then Eq. (4.2.37) can be expressed as follows:

$$\mathbf{y} = H\mathbf{x} + \boldsymbol{\epsilon} \quad (4.2.39)$$

where $\mathbf{y}$ is an $m \times 1$ vector, $\mathbf{x}$ is an $n \times 1$ vector, $\boldsymbol{\epsilon}$ is an $m \times 1$ vector, H is an $m \times n$ mapping matrix, where $m = p \times \ell$ is the total number of observations. If p or ℓ is sufficiently large, the essential condition $m > n$ is satisfied. However, we are still faced with m unknown observation errors resulting in $m + n$ total unknowns and only m equations. The least squares criterion provides us with conditions on the m observation errors that allow a solution for the n state variables, $\mathbf{x}$, at the epoch time t_k .

4.3 THE LEAST SQUARES SOLUTION

$$\mathbf{y} = H\mathbf{x} + \boldsymbol{\epsilon} \quad (4.2.39)$$

$$J(\mathbf{x}) = 1/2\boldsymbol{\epsilon}^T \boldsymbol{\epsilon}. \quad (4.3.1)$$

$$J(\mathbf{x}) = 1/2\boldsymbol{\epsilon}^T \boldsymbol{\epsilon} = \sum_{i=1}^{\ell} 1/2\boldsymbol{\epsilon}_i^T \boldsymbol{\epsilon}_i = 1/2(\mathbf{y} - H\mathbf{x})^T (\mathbf{y} - H\mathbf{x}) \quad (4.3.2)$$

Note that Eq. (4.3.2) is a quadratic function of $\mathbf{x}$, and as a consequence the expression will have a unique minima when (see Appendix B, Eq. (B.8.2))

$$\frac{\partial J}{\partial \mathbf{x}} = 0, \quad \text{and} \quad \delta \mathbf{x}^T \frac{\partial^2 J}{\partial \mathbf{x}^2} \delta \mathbf{x} > 0$$

for all $\delta \mathbf{x} \neq 0$. The second condition implies that the symmetric matrix

$$\frac{\partial^2 J}{\partial \mathbf{x}^2}$$

is positive definite.

Carrying out the first operation on Eq. (4.3.2) (see Appendix B, Eq. (B.7.3)) yields

$$\frac{\partial J}{\partial \mathbf{x}} = 0 = -(\mathbf{y} - H\mathbf{x})^T H = -H^T (\mathbf{y} - H\mathbf{x}) \quad (4.3.3)$$

The value of $\mathbf{x}$ that satisfies Eq. (4.3.3) will be the best estimate of $\mathbf{x}$, which we will call $\hat{\mathbf{x}}$. Hence,

$$(H^T H)\hat{\mathbf{x}} = H^T \mathbf{y} \quad (4.3.4)$$

These are the normal equations.

Also, from Eqs. (4.3.3) and (B.7.3) it follows that

$$\frac{\partial^2 J}{\partial \mathbf{x}^2} = H^T H \quad (4.3.5)$$

which will be positive definite if H is full rank.

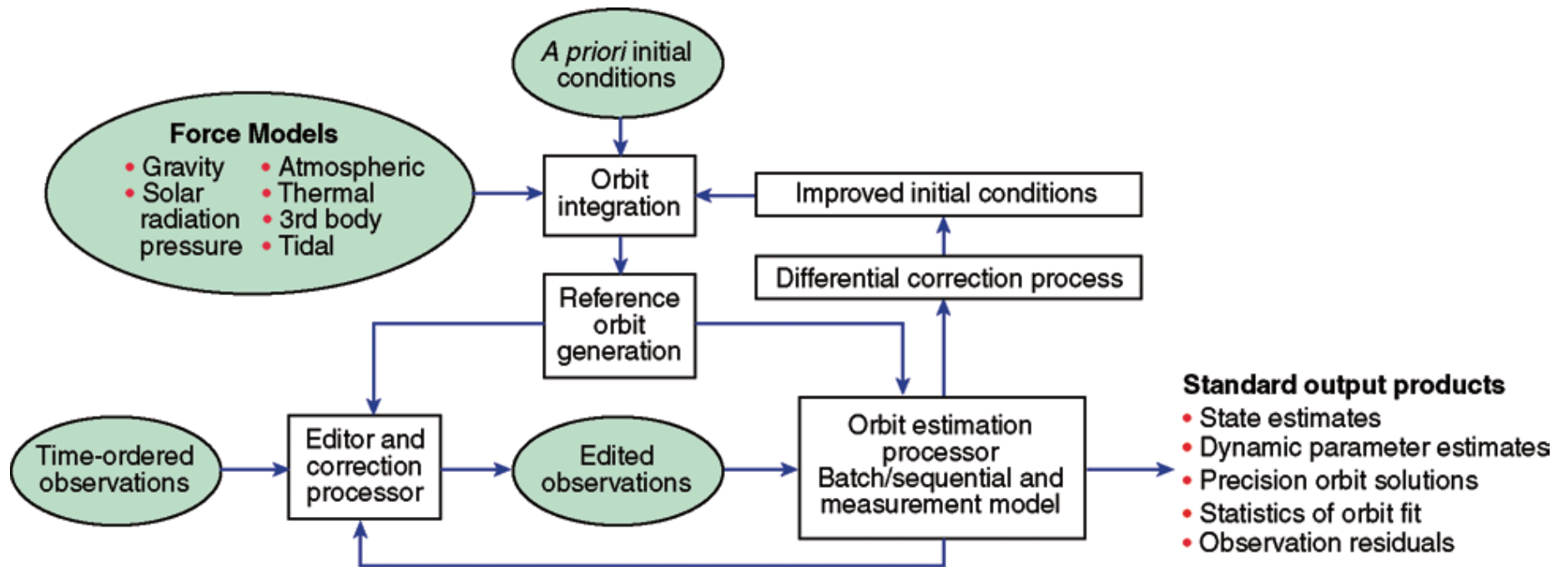
Equation (4.3.4) is referred to as the normal equation, and $(H^T H)$ is referred to as the *normal matrix*. Note that the matrix $H^T H$ is an $n \times n$ symmetric matrix, and if this matrix is positive definite (H is rank n) then the solution for the best estimate of $\mathbf{x}$ is given by

$$\hat{\mathbf{x}} = (H^T H)^{-1} H^T \mathbf{y}. \quad (4.3.6)$$

Equation (4.3.6) is the well-known least squares solution for the best estimate of $\mathbf{x}$ given the linear observation state relationship expressed by Eq. (4.2.39). With the observations, $\mathbf{y}$, and a specified value of $\hat{\mathbf{x}}$, the value for the best estimate of the observation errors, $\hat{\epsilon}$, can be computed from Eq. (4.2.39) as

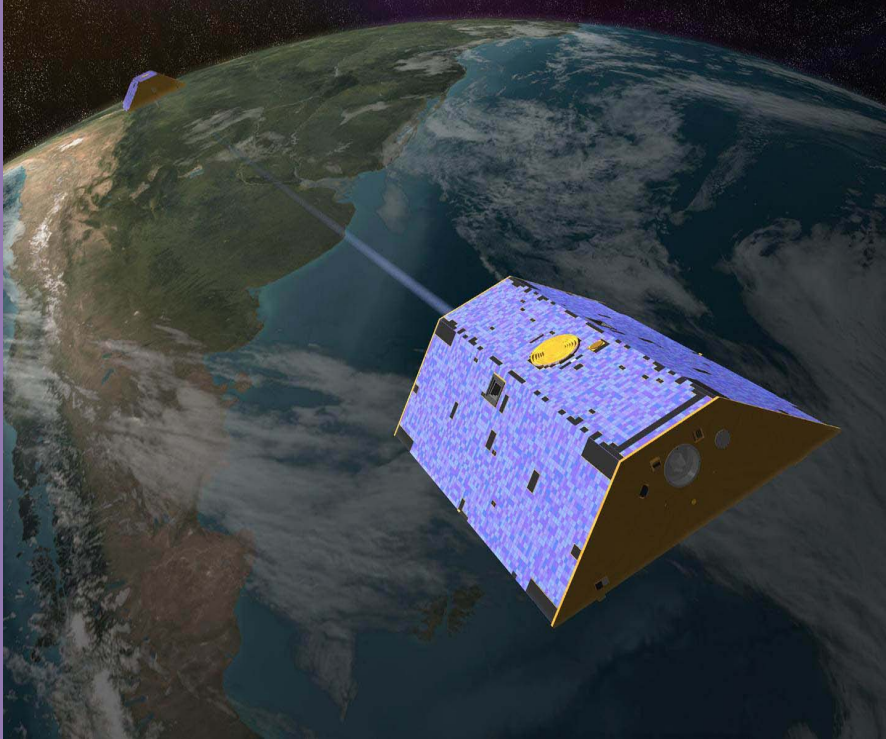
$$\hat{\epsilon} = \mathbf{y} - H\hat{\mathbf{x}}. \quad (4.3.7)$$

General POD Elements

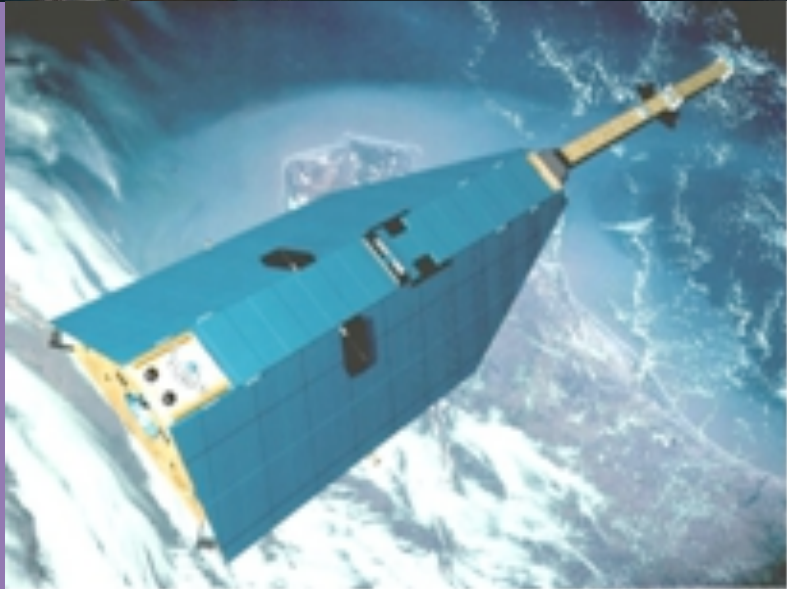
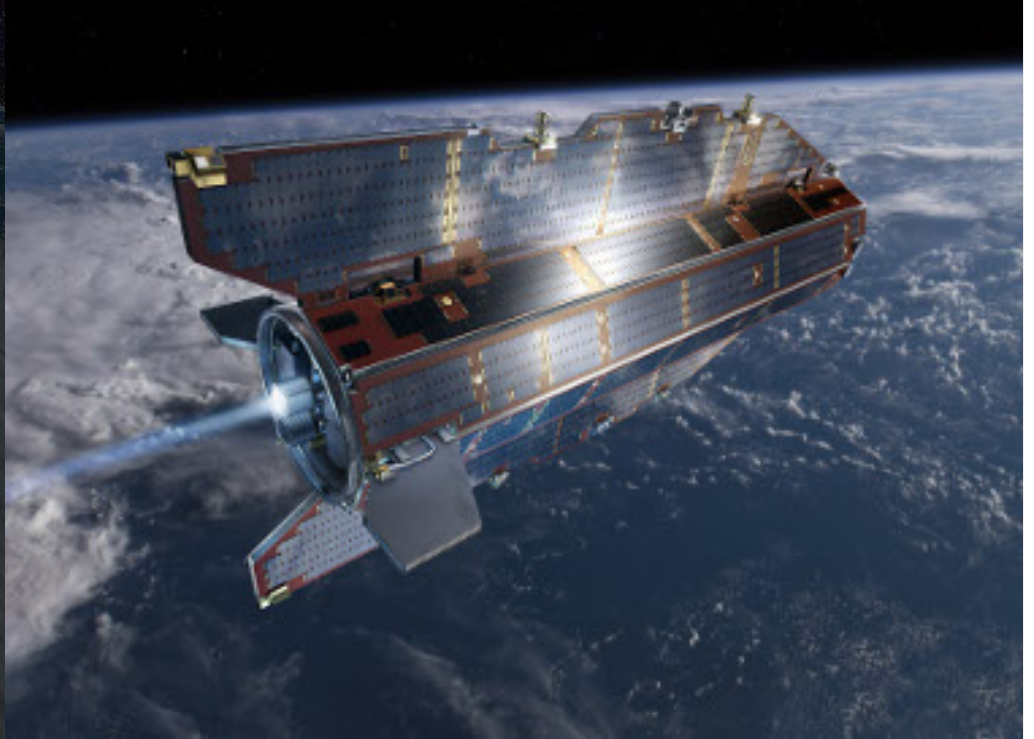


Gravity Model Determination

GRACE (2002)

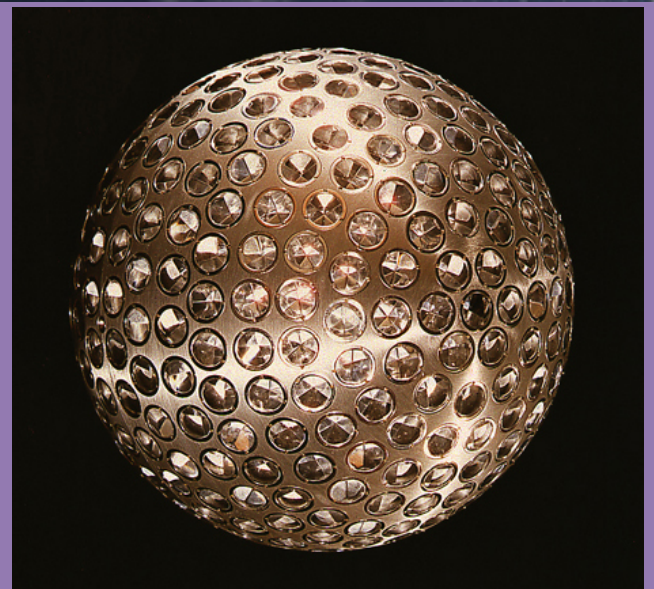


GOCE (2009)



CHAMP (2000)

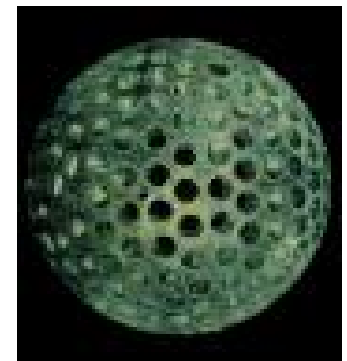
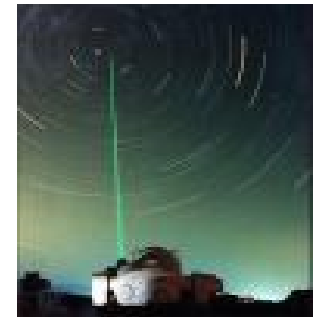
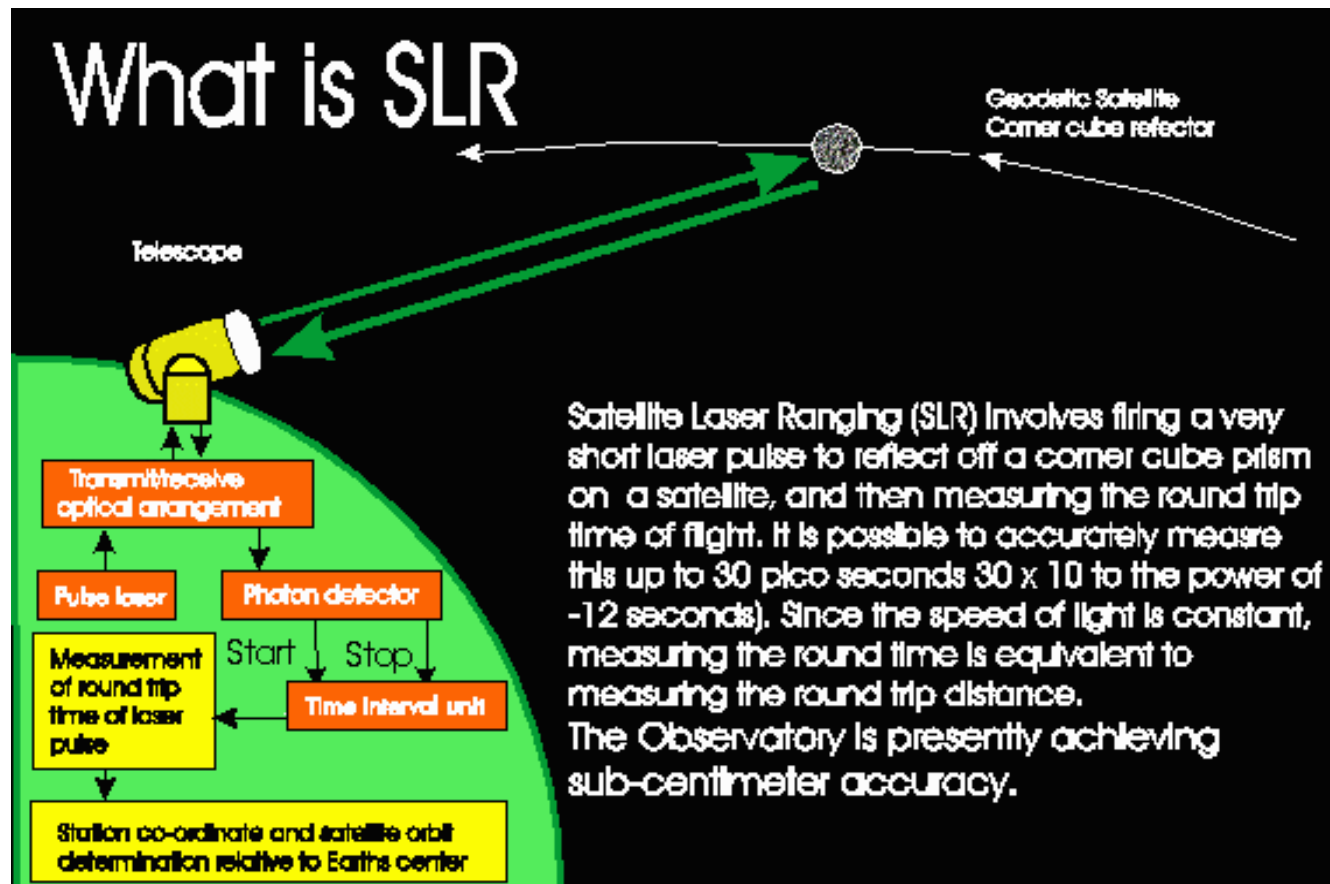
LAGEOS(1978)



Gravity

Time Variable Gravity Model

Satellite Laser Ranging

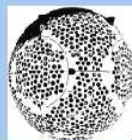




LAGEOS-1,2



Starlette,Stella



Etalon-1,2



Ajisai



GFZ-1



GFO-1



ADEOS/RIS



ERS-1,2



TOPEX/Poseidon

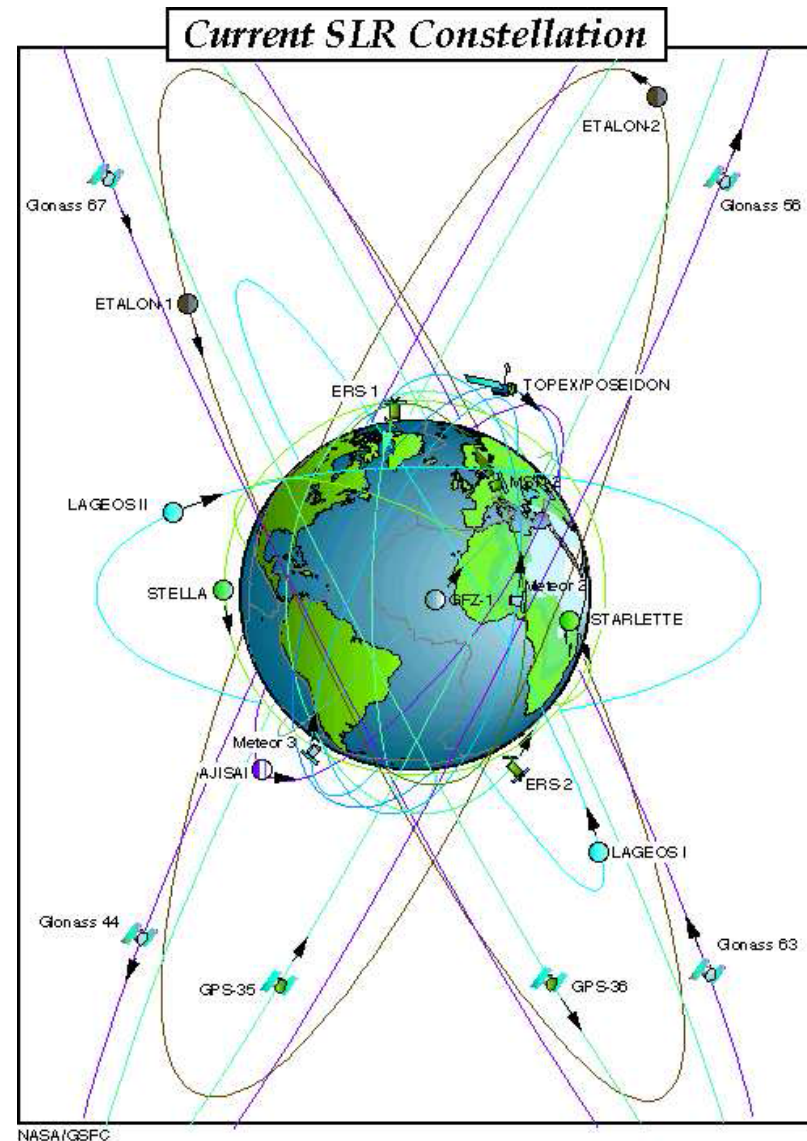


GPS-35,36

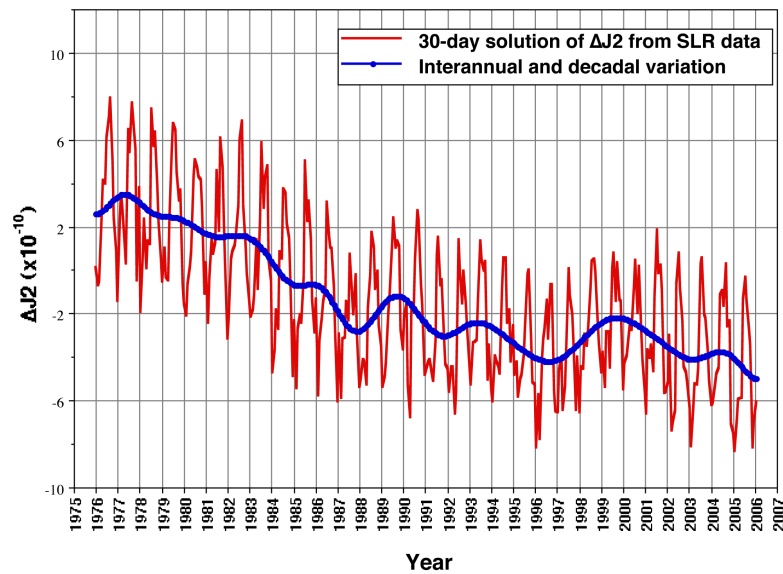


Glonass-63,67

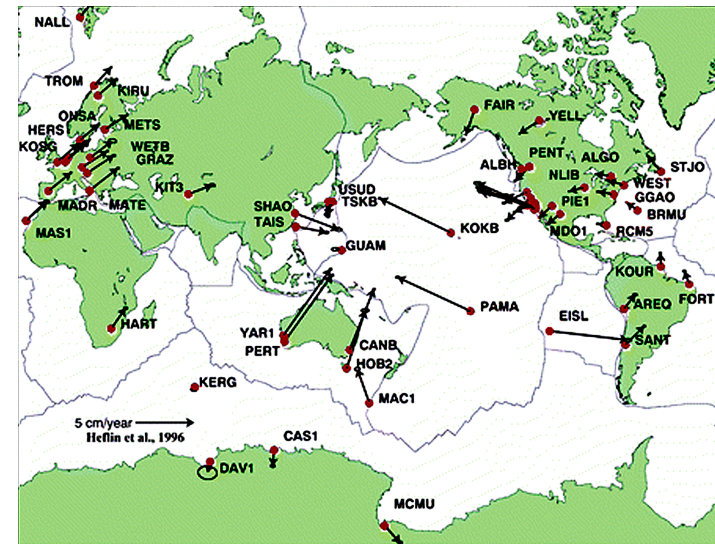
SLR Satellite Constellation



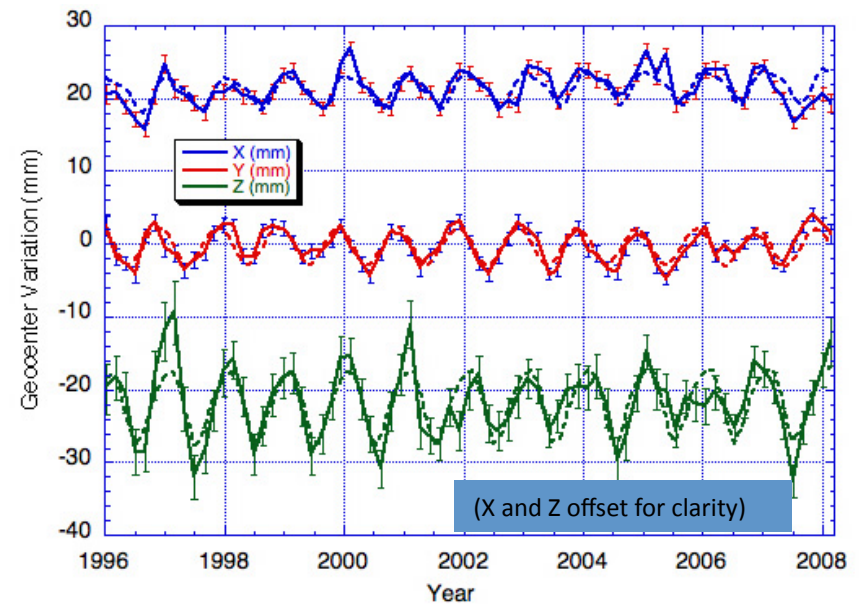
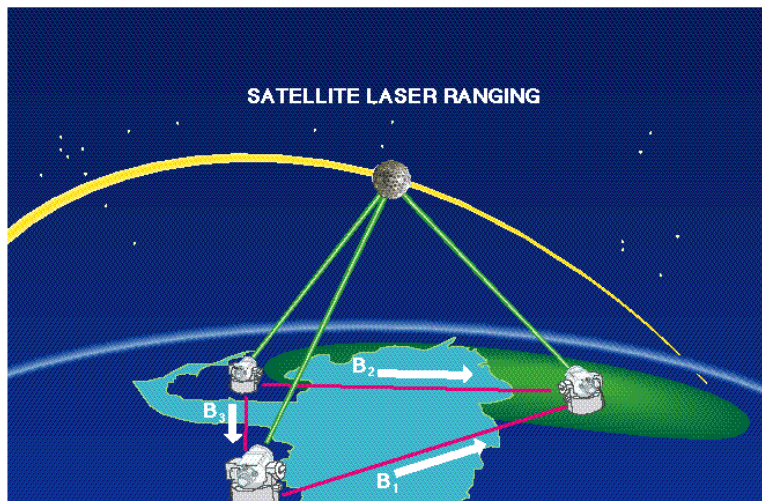
Examples of Earth Science from SLR



Long-term trends in J_2 due to PGR as well as mass redistribution due to water exchange between and within the cryosphere and hydrosphere

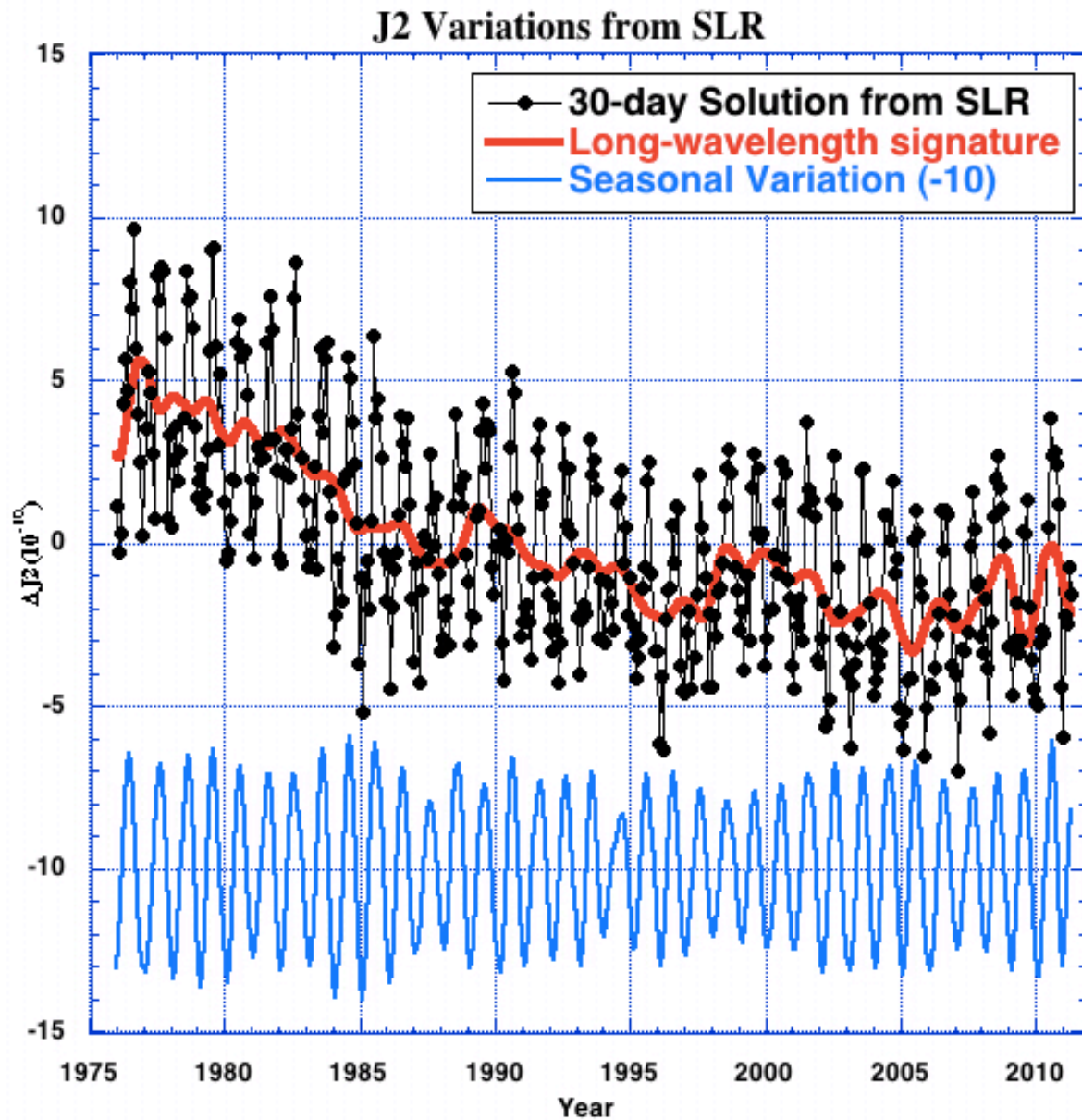


Tectonic plate motions observed by space techniques

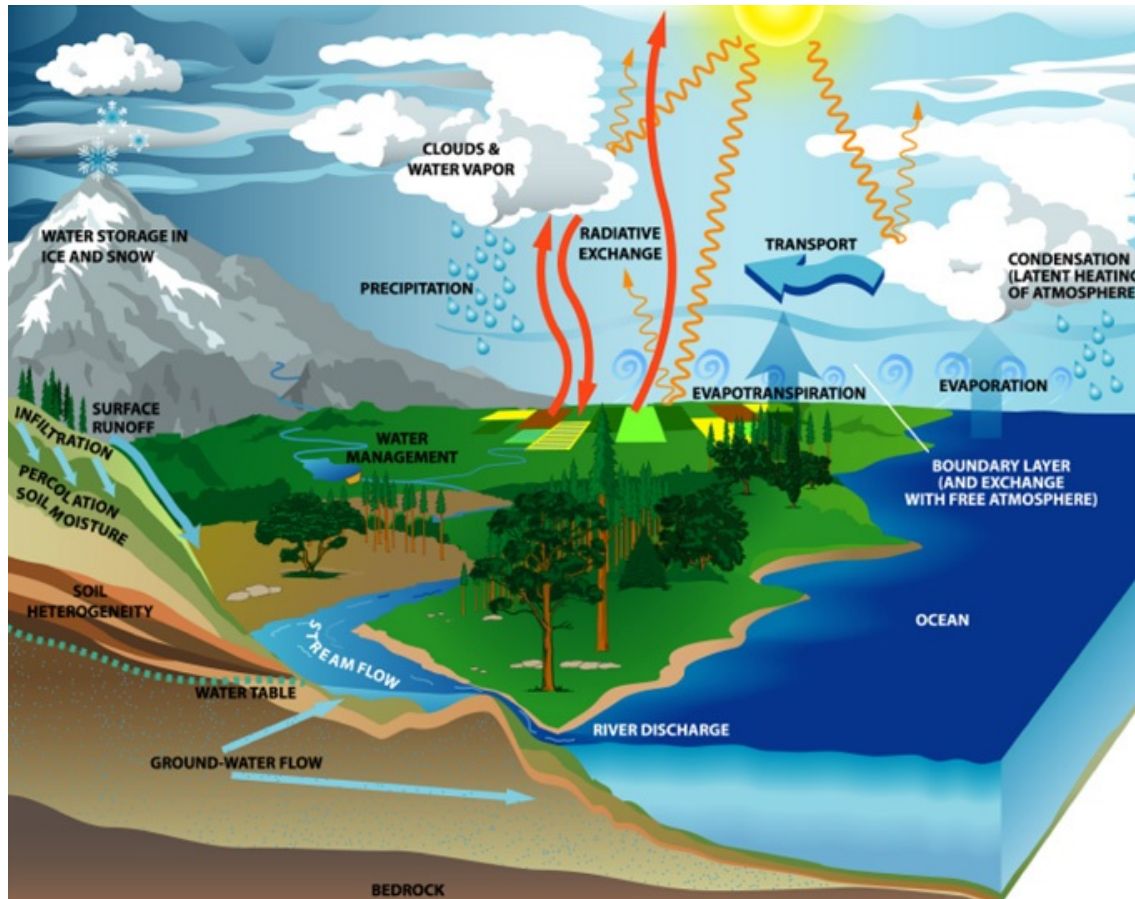


Geocenter motion of just a few mm observable with SLR

SLR and Global Mass Variation



Mission Science Objective



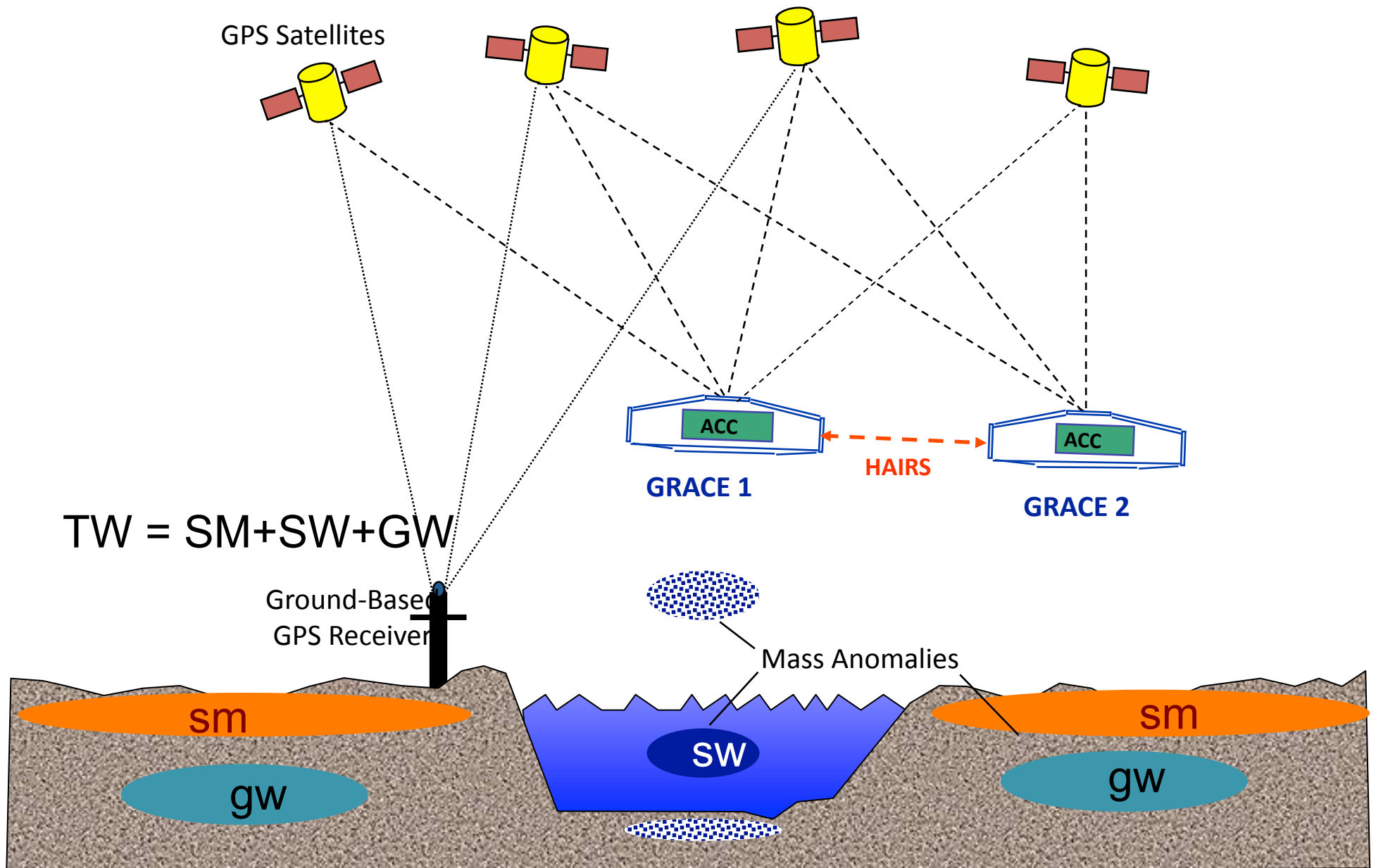
Measure Global Gravity to Determine Mean Global Mass Distribution and the Temporal Variations

- GRACE measures the change in all forms of the water stored on land after precipitation has been stored as snow, infiltrated into the ground, evaporated, or left the basin as streamflow

- Water balance accounts for the deposition, outflow and storage change

$$\text{Storage Change} = \text{Inflow (Precipitation)} - \text{Outflow (Evaporation + Streamflow)}$$

Grace Mission Concept



GRACE MEASUREMENT CONCEPT

Sample the Earth's Surface ~ every 30 days (GGM05M)

Convert 30 day data set into a gravity model representative of mass distribution during each sample interval

- Six hour global output from ECMWF Meteorological Model to obtain rapid atmospheric mass transport
- Surface pressure and winds from ECMWF Models are used to force Ocean Model to obtain high frequency Ocean response to atmospheric forcing

Average ~ 125 monthly solutions(>11.5 year mission) used to obtain long term mean (GGM05S)

Difference monthly solutions from the long term mean to obtain monthly mass variability

Combine GGM05S with GOCE and DTU10 (GGM05C)

RL05 Solution Overview

Status of the RL05 Data Release

Solutions from January 2003 – May 2014 have been released

The Level-2 data for 2002 was released on December 6, 2013

Continued effort to Complete Mean Field

Continued Effort to Calibrate the Error Covariance

Some Observations

Factor of two improvement in solution accuracy, overall

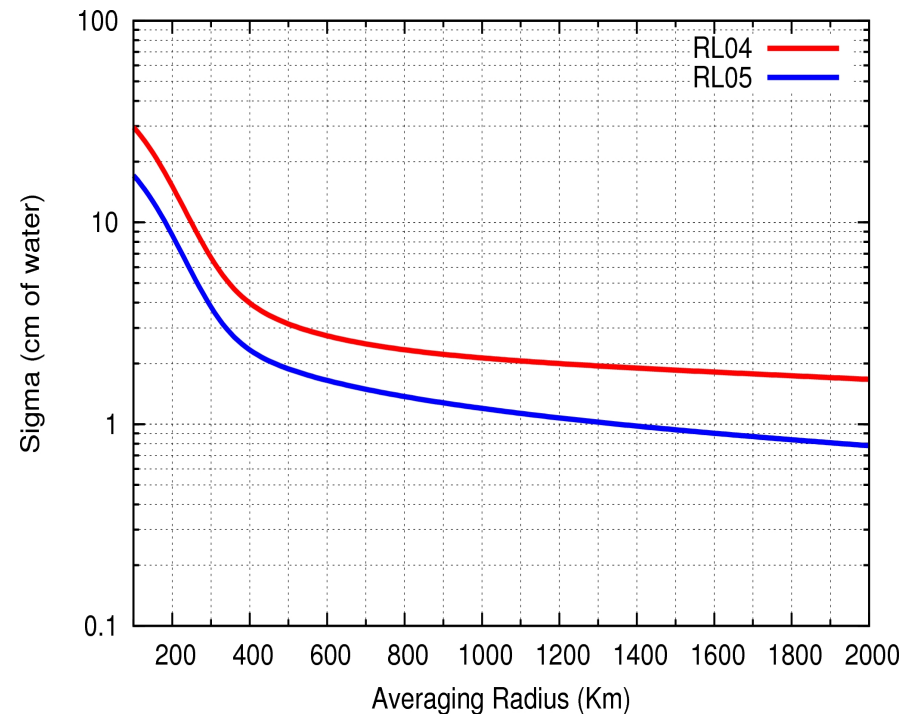
For spatial scales < 500 km, the solutions are more noisy after 2010 when thermal control was relaxed

Vast majority of users should see no effect on science results after 2010

Issues

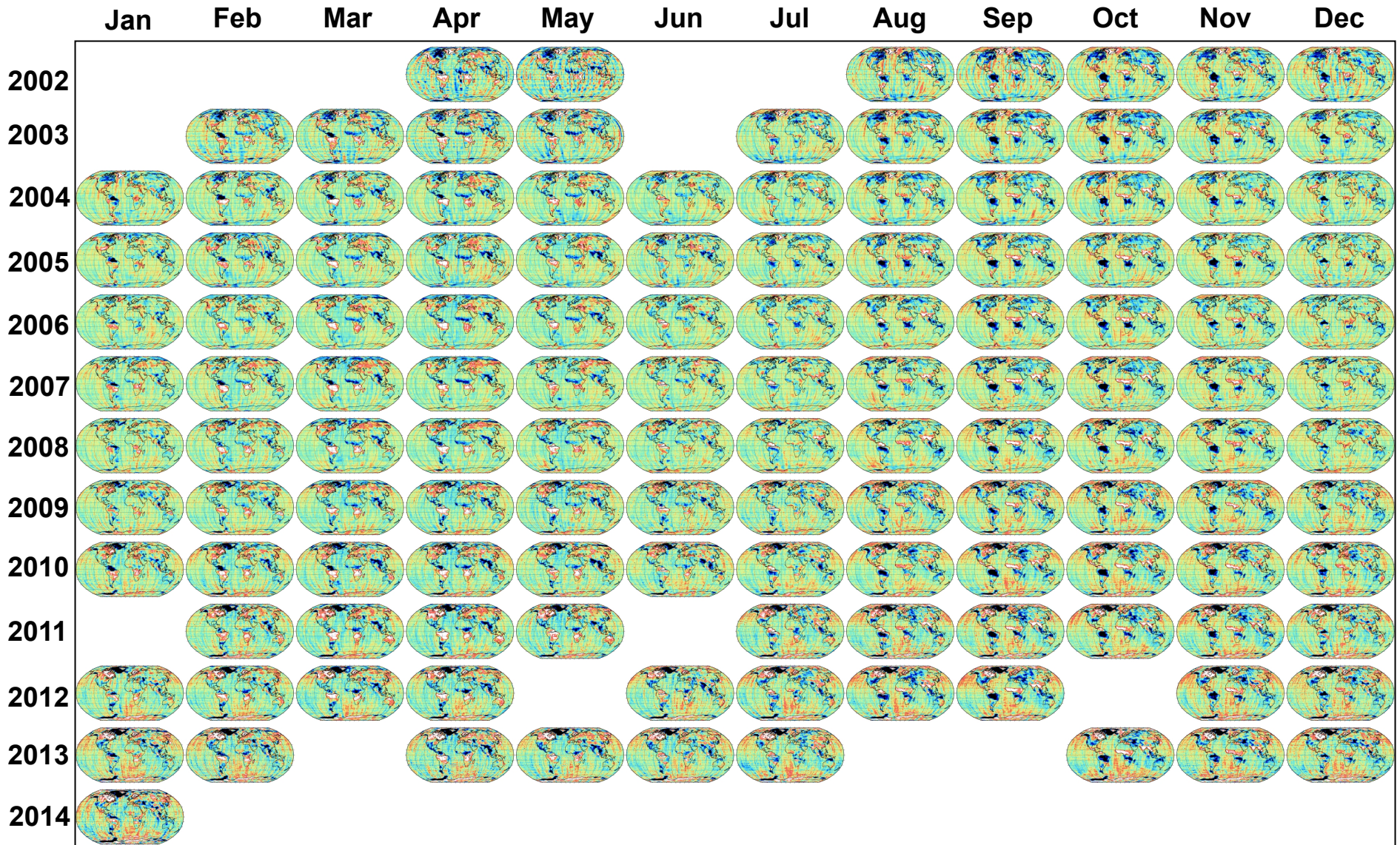
Slope differences between center solutions was resolved

Differences in model resolution (60 vs 90)

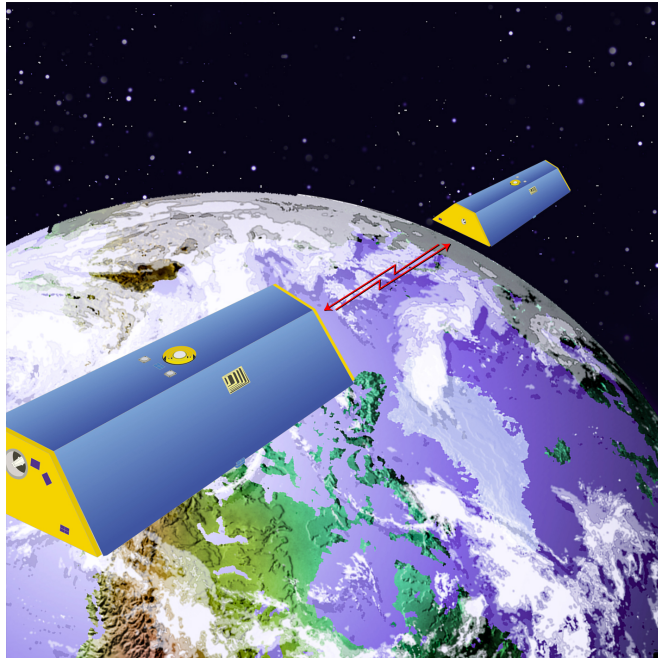


CSR RL05

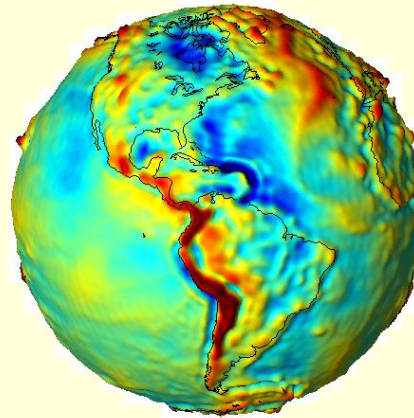
Over 12 Years in Orbit – 133 Global Gravity Solutions



Global gravity models from space



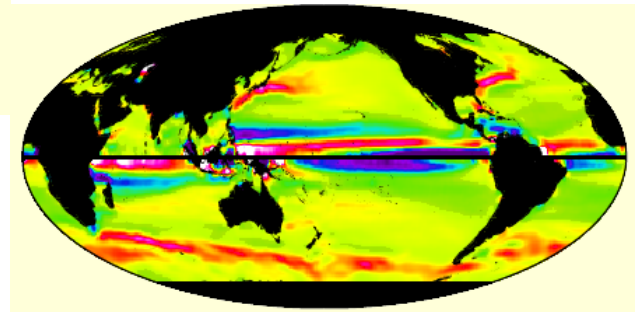
GRACE (Gravity Recovery and Climate Experiment) launched 2002



Global gravity anomalies from GRACE

Global mean gravity field model improved by orders of magnitude

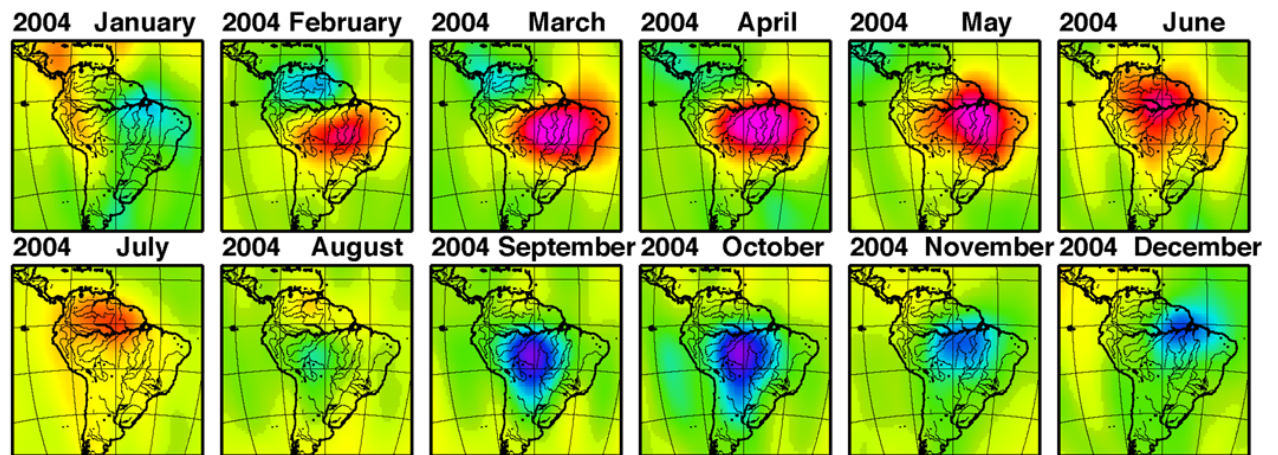
Absolute ocean surface currents can be clearly resolved with unprecedented accuracy



Zonal ocean circulation inferred from GRACE geoid and altimetry-based mean sea surface

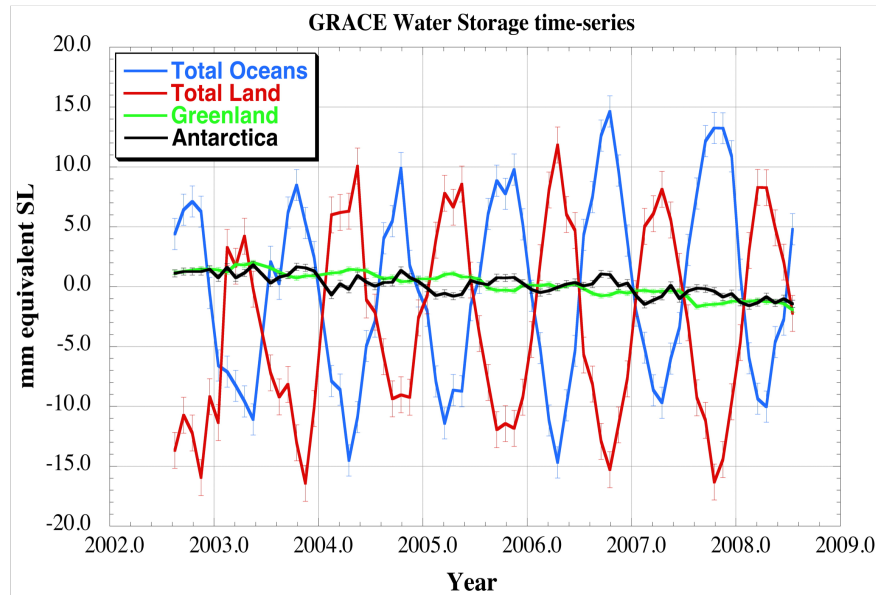
Temporal gravity changes observed by GRACE

A clear separation can be observed between the large Amazon watershed and the smaller watersheds to the north, indicating that basin-scale variability is being resolved



Mass variations are observed in Amazon basin with ~400 km resolution (range is from -12 to +12 cm of water)

GRACE Mass Estimate



Trends (mm/yr)

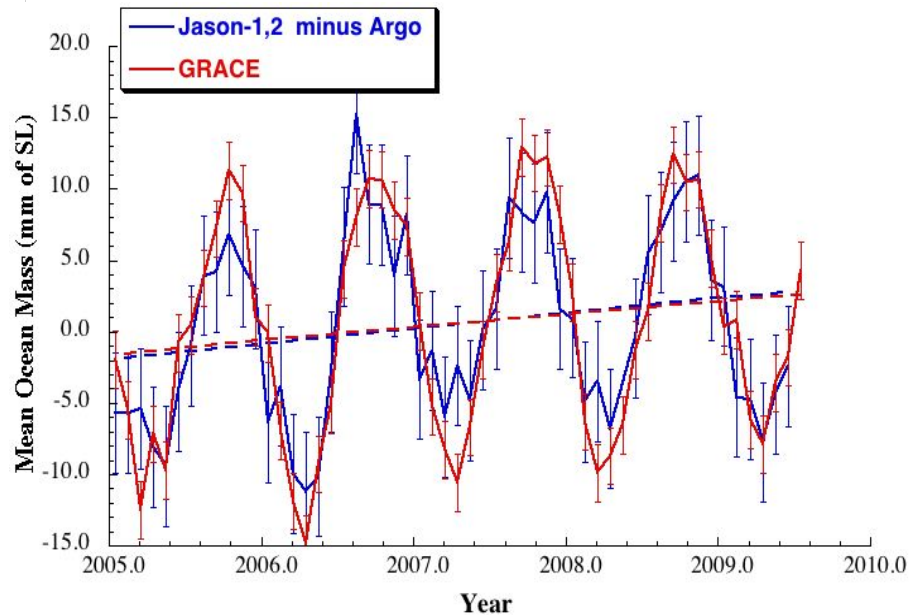
Ocean = 1.2 ± 0.3

Land = 0.3 ± 0.5

Greenland = -0.60 ± 0.1

Antarctica = -0.40 ± 0.2

Famiglietti, 2009

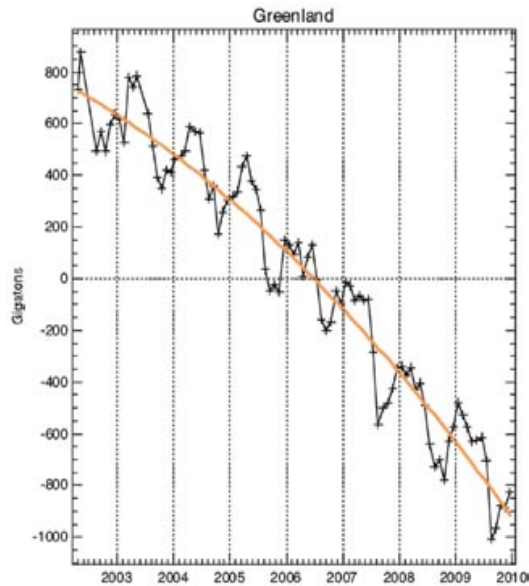


GRACE/Jason/Argo Closure

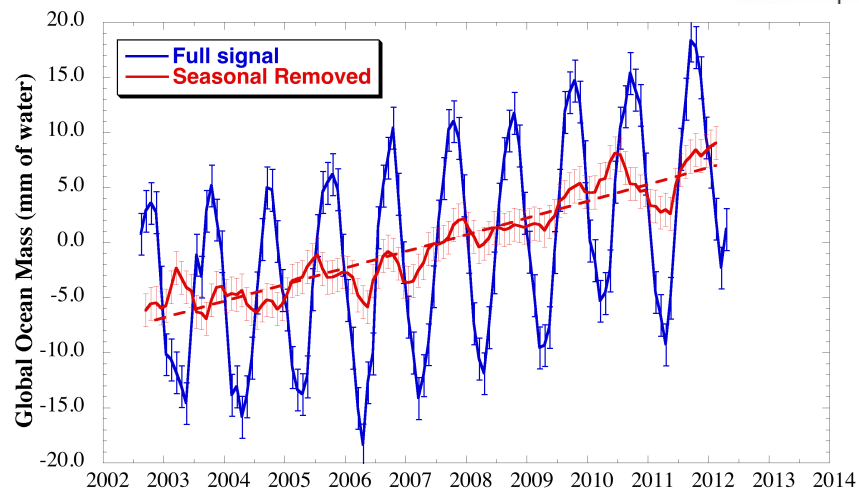
Grace Trend(2003-2009.5) = 1.3 ± 0.8 mm/yr

Chambers, 2009

Global Sea Level Change

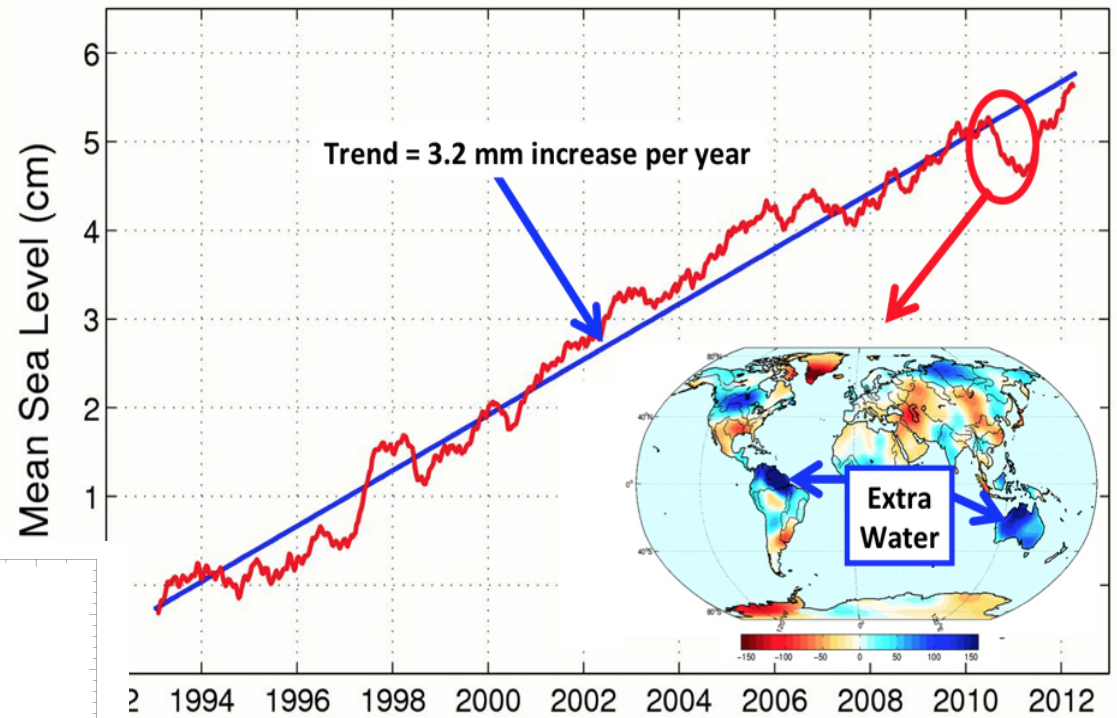


John Wahr, et al



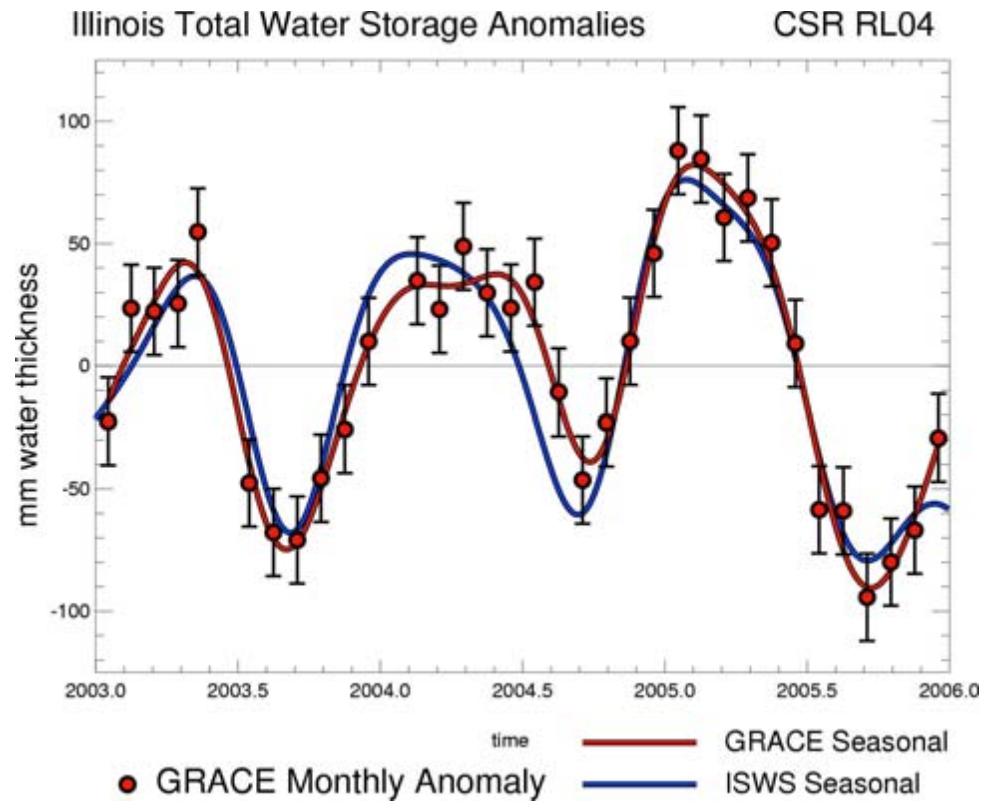
Error bars are 1 standard error.

D. Chambers, 2013



Carmine Boening, et al, 2013

Comparison of Illinois Water Storage Anomalies

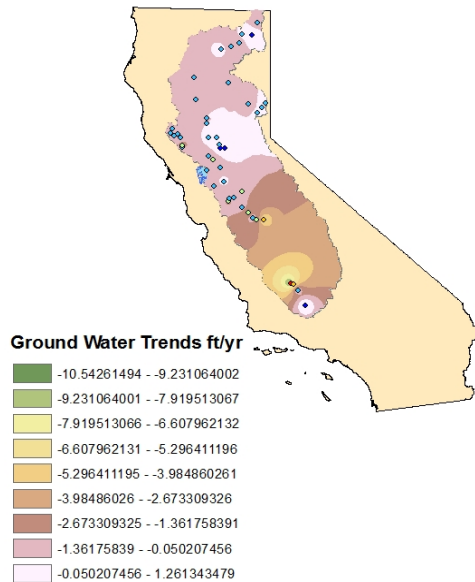


Area of 280,000 km²

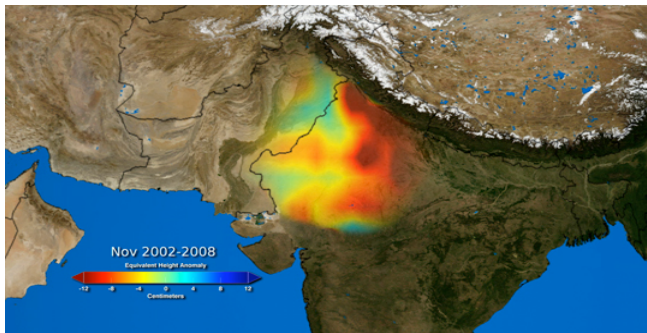


(From Swenson, 2007)

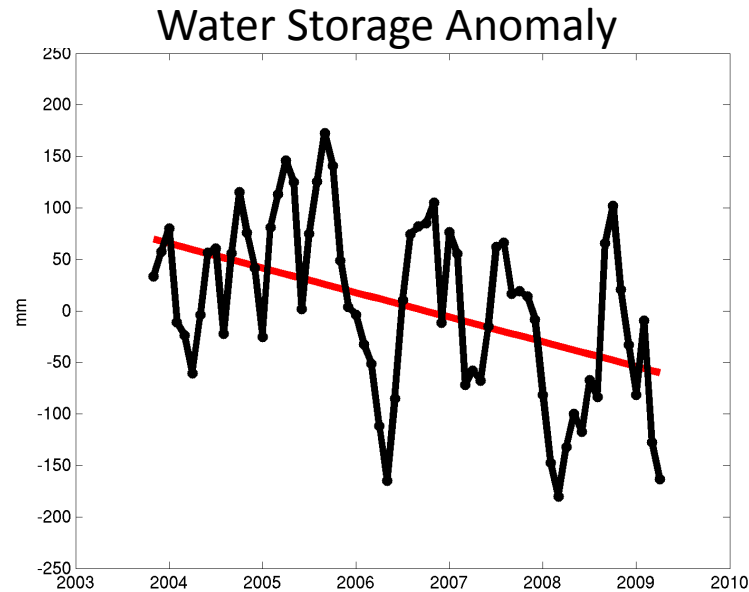
GRACE DETECTS UNSUSTAINABLE GROUNDWATER LOSS



Observed trends in groundwater levels, October, 2003 – March, 2009

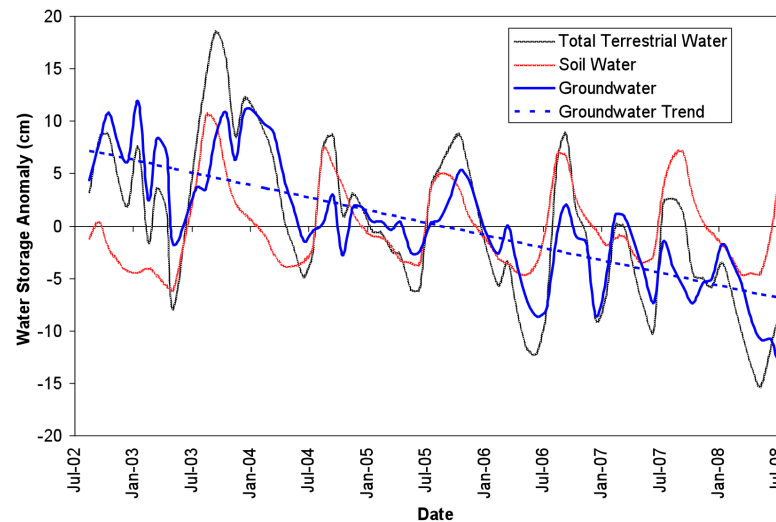


Pattern of groundwater depletion in NW India



Drawdown by 31 km^3
(= 1 Lake Mead)
in 66 months

Famiglietti et al., 2009

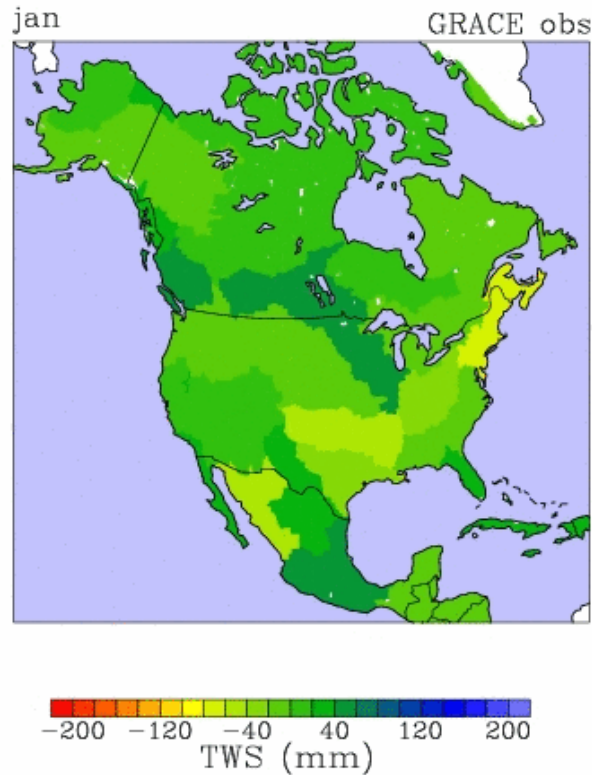


Loss of 109 km^3
(3 Lake Meads)
over 72 months

Rodell et al., 2009

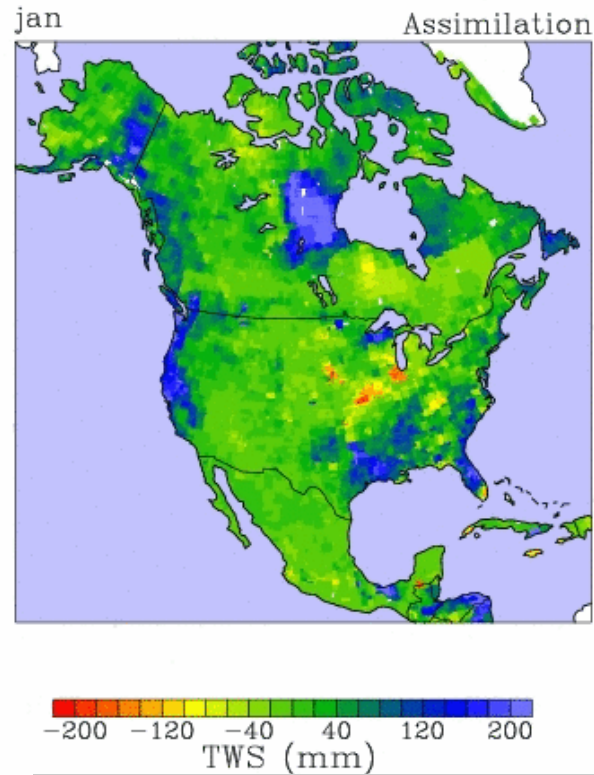
GRACE Data Assimilation

GRACE water storage, mm
January-December 2003 loop



From scales useful for water cycle and climate studies...

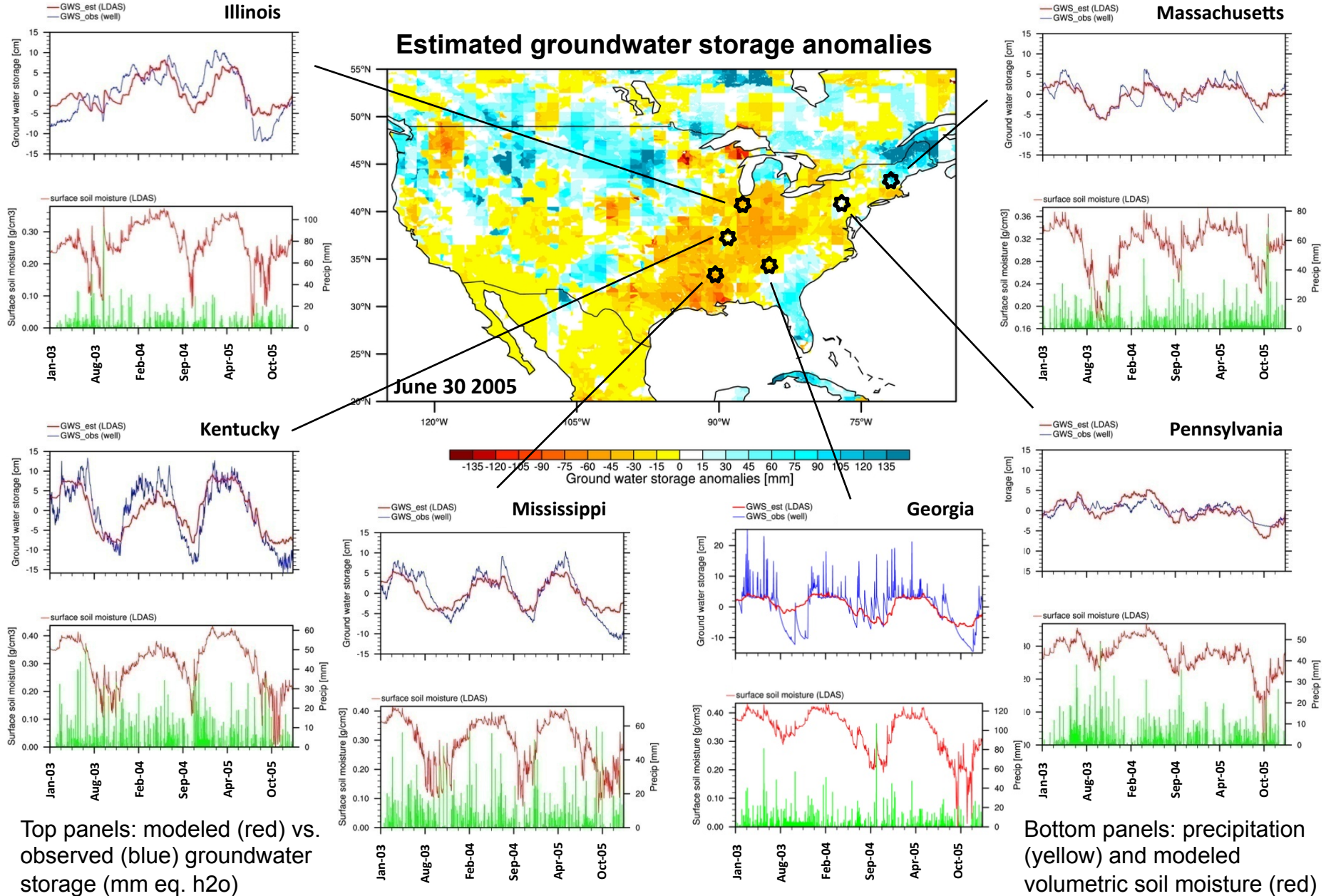
Model assimilated water storage, mm
January-December 2003 loop



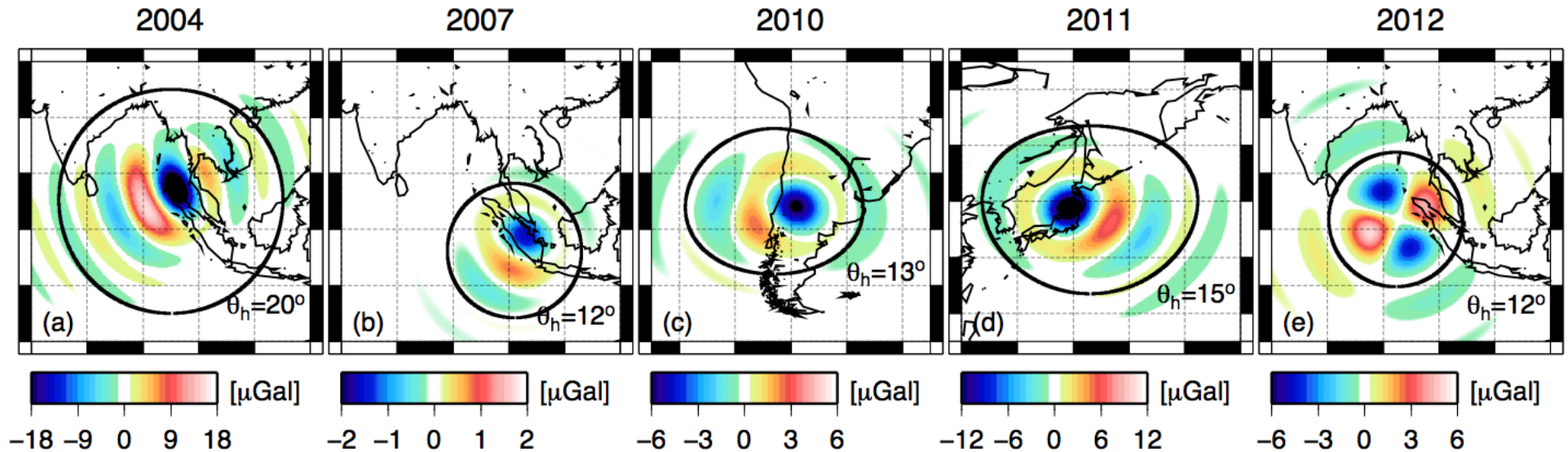
Monthly anomalies (deviations from the 2003 mean) of terrestrial water storage (sum of groundwater, soil moisture, snow, and surface water) as an equivalent layer of water. Updated from Zaitchik, Rodell, and Reichle, *J. Hydromet.*, 2008.

To scales needed for water resources and agricultural applications

GRACE Assimilated vs. in situ Groundwater



GRACE Gravity Changes for Recent Great Earthquakes



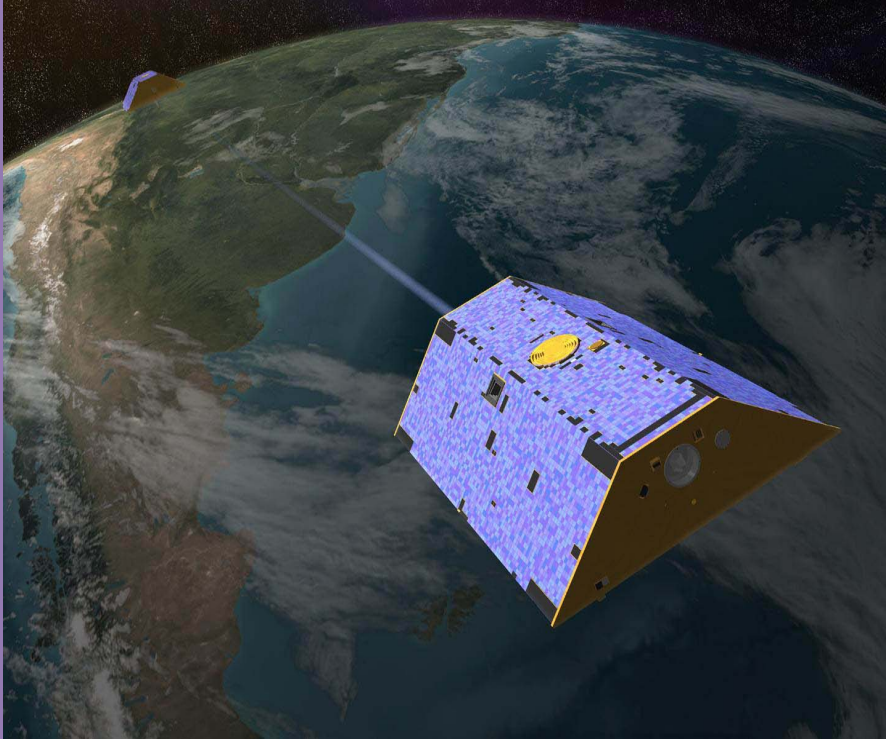
Riccardo Riva (TU Delft), [Shin-Chan Han](#) (NASA), Jeanne Sauber (NASA),

Emile Okal (Northwestern Univ), Fred Pollitz (USGS)

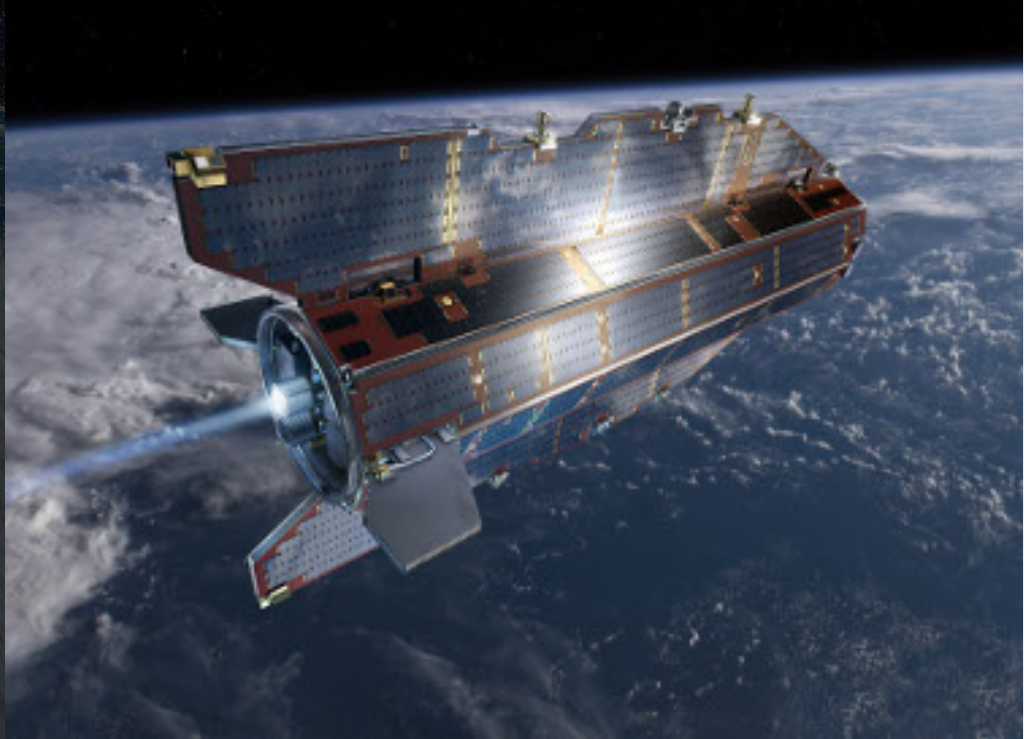
AGU Fall Meeting, San Francisco, California, December 9-13, 2013

Gravity

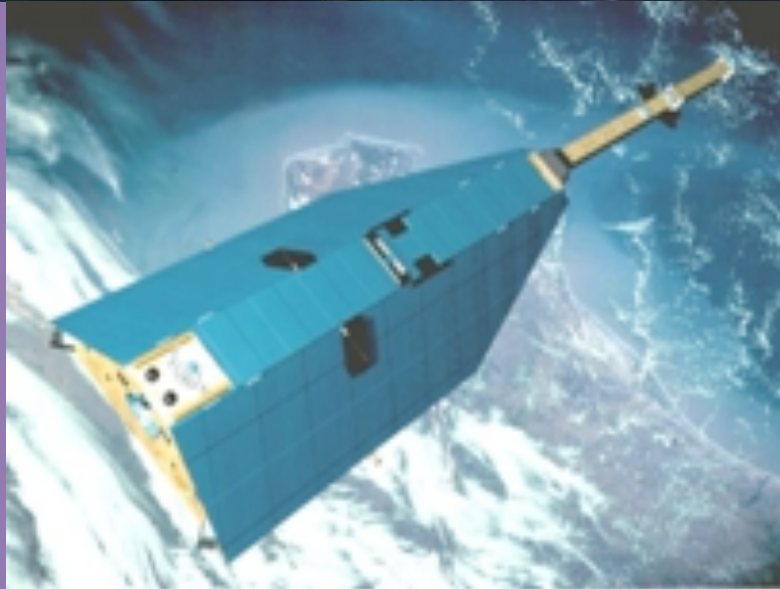
Global Mean Gravity Model



GRACE (2002)



GOCE (2009)

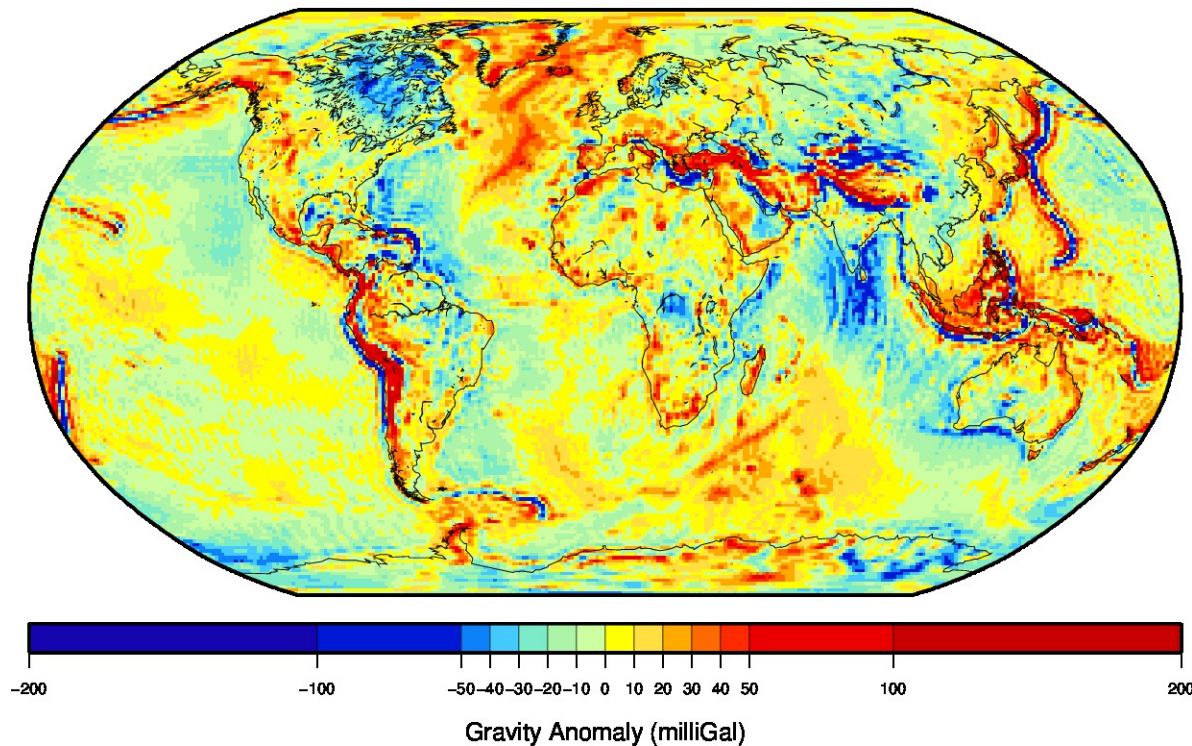


CHAMP (2000)

GGM05S

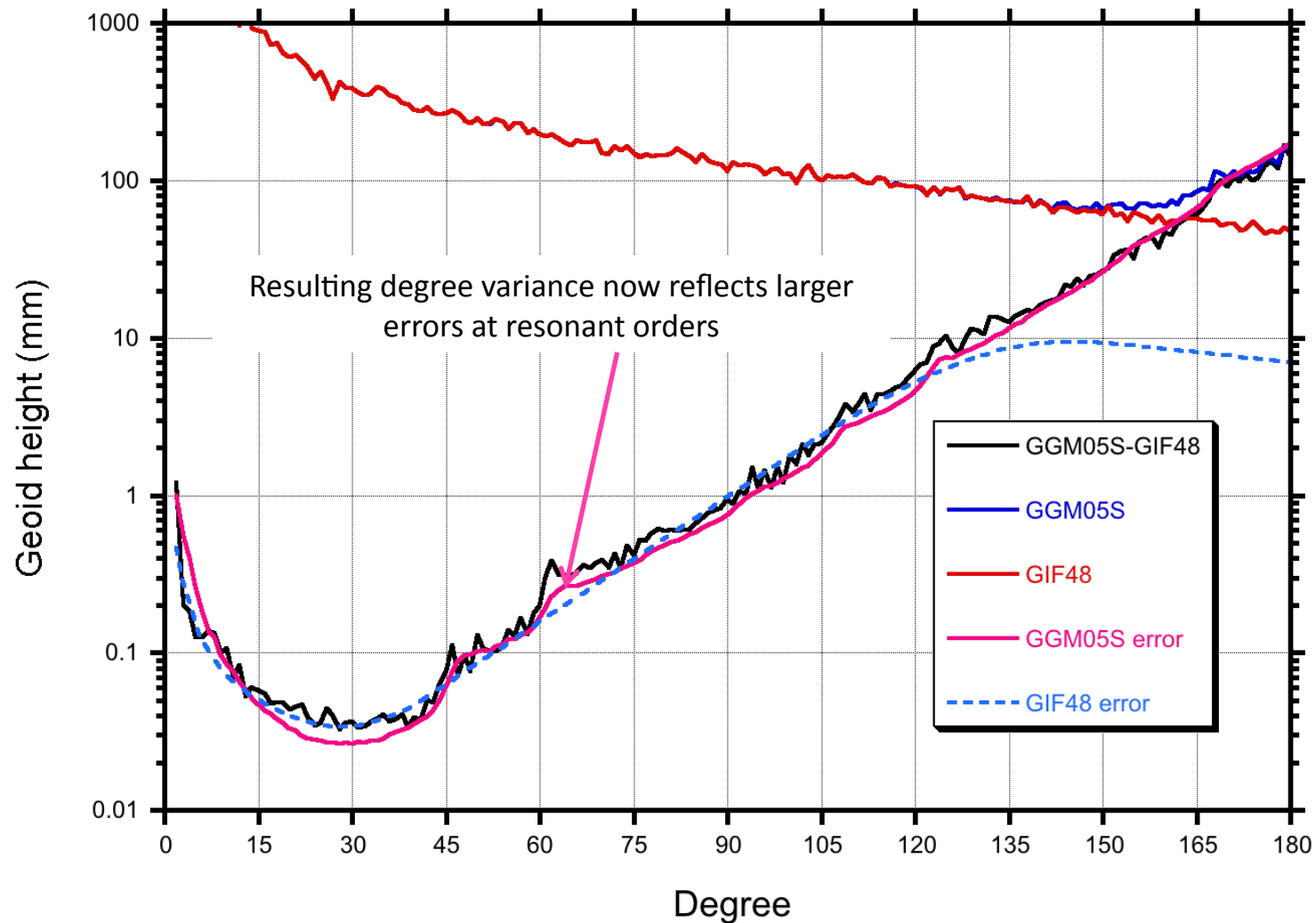
Ten-year (March 2003 to April 2013) combination to degree/order 180, of GRACE monthly estimates (no Kaula constraint)

GGM05SNormal.n150.00.km



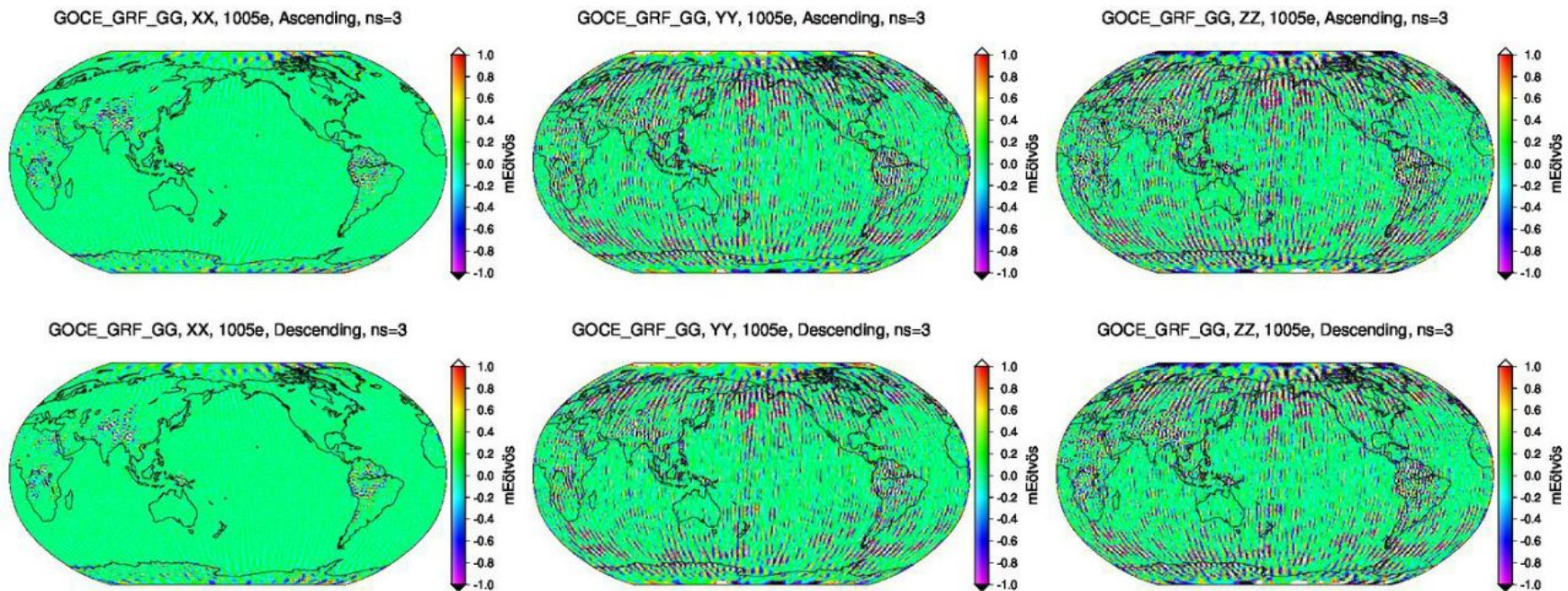
GGM05S Error Calibration

Degree- and order-dependent calibration better reflects the nature of the errors



GOCE contribution

Maps show GOCE data residual relative to Gf48 where terrestrial gravity was poor/unavailable; and shows the striations from GRACE

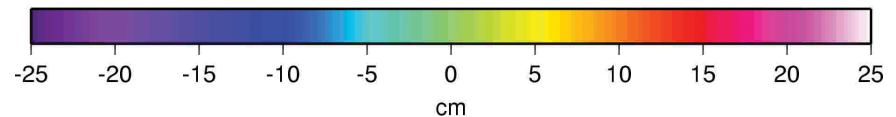
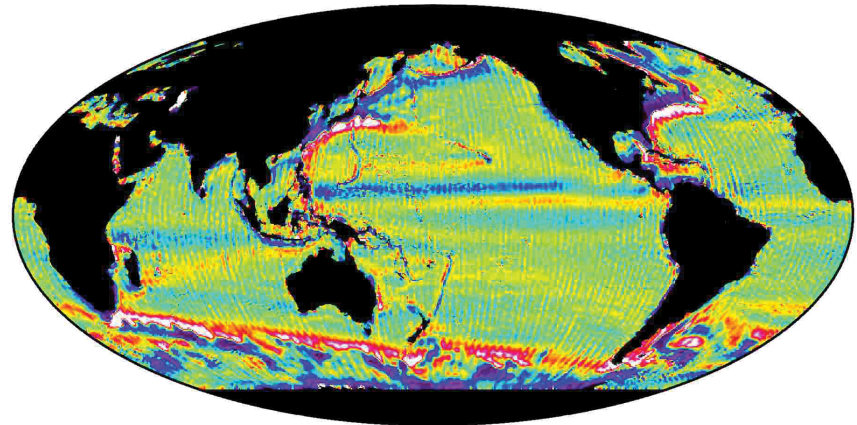
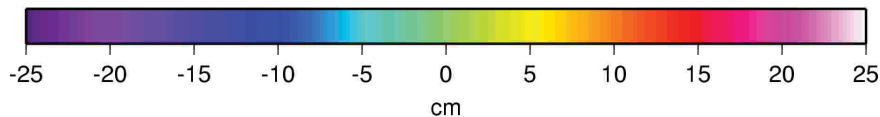
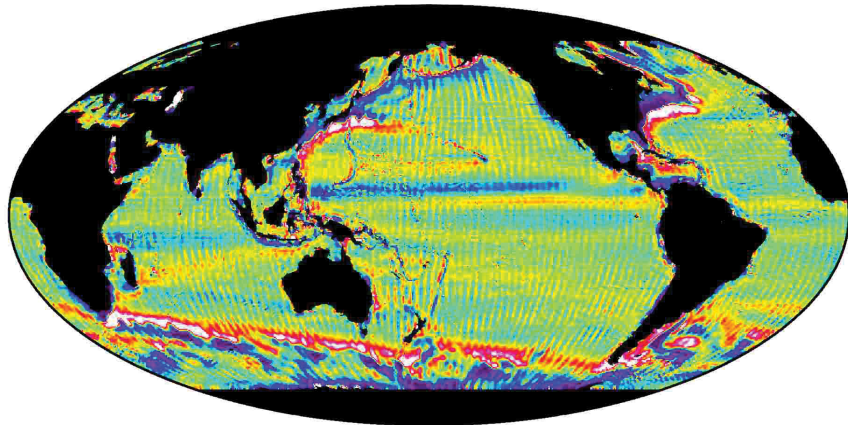


Low residuals in XX direction (first column) show analogy between the GOCE gravity gradients and GRACE data in the along-track directions.

GIF48 vs GIF54

66 selected months of RL04 combined with DTU10 (variance-only, degree-dependent calibration of GRACE)

~10 years of RL05 (GGM05S) combined with DTU10 (full covariance, degree and order dependent calibration of GRACE)



‘Striations’ are visibly reduced with improved GRACE information and better GRACE error calibration

Striations are difficult to eliminate entirely without significantly upweighing surface gravity relative to GRACE (e.g., EGM2008)

We expect GOCE to flatten the striations, as these seem to be dominantly from terms (particularly the near-sectorials) above degree 130 or so

GOCE Analysis

Status:

Pre-processing, examination of residuals, and preliminary data screening has been completed for data spanning Nov 2009 to Jan 2013

Challenges:

Band-pass filtering strategy: adopted (10-50 mHz)

Data screening

'Polar gap' problem: addressed by filling with GGM05S
using synthetic gradients at altitude

Relative weighting of GRACE, GOCE and DTU10



Preliminary results: Based on 715 days of XX, YY, ZZ and XZ gradients + polar gap fill (to degree/order 240x240)

Data evaluation and editing is near completion

First 61 days of XX, YY, ZZ and XZ combination have been evaluated;
indicating that all components are useful to varying degrees

YY is more noisy than the other components for the full 715 day

Preliminary solutions are under evaluation.

Status

GRACE has been in orbit for ~ 12.33 years and **127 monthly gravity model** solutions have been released.

GRACE measurements have improved the understanding of the Earth Dynamic System's **secular, seasonal and inter-annual signals** (Recognized as a Climate Mission) and has important contributions to the determination of **the mean gravity model**

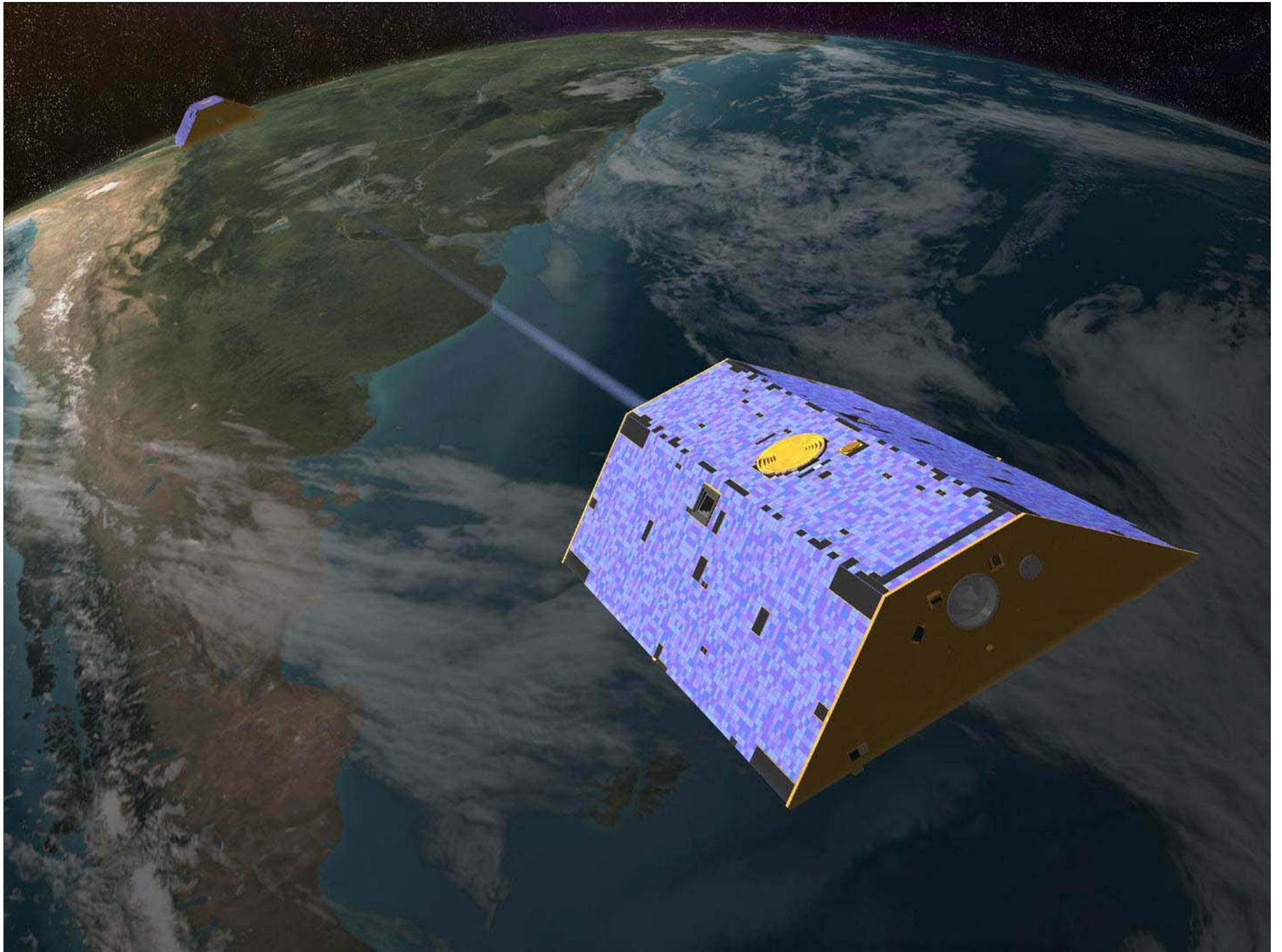
Application **User Community is developing**

Continued scientific advances are expected from **improved spatial-temporal resolution obtained with RL-05 gravity models**

GRACE **instruments performing nominally**,
End of mission likely due to battery problems or fuel consumption

Project focus on extending the mission to overlap with GRACE-FO
Battery friendly operations mode
Increased Level 2 Processing effort to maintain scientific accuracy
Bridging concepts under study for use in loss of mission capability

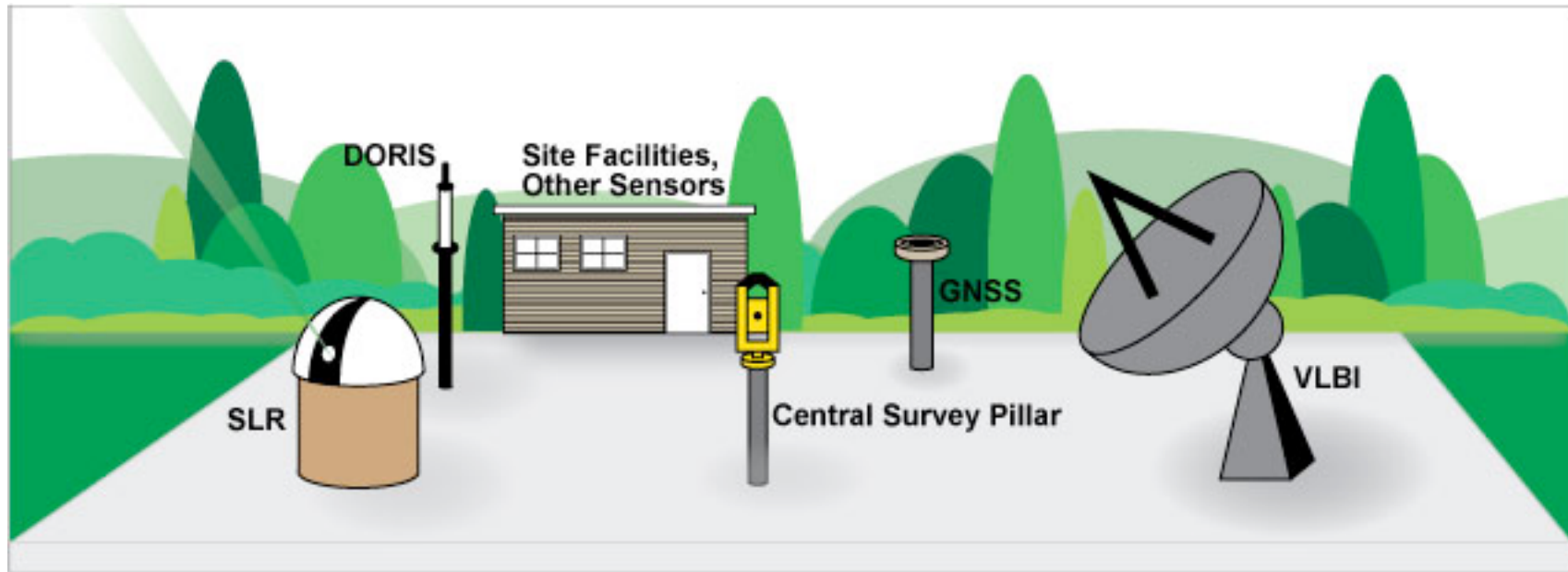
2014 GRACE Science Team Meeting in Potsdam, Germany– Sept 21-25, 2014



END

IGS Global Geodetic Observing System (GGOS) Core Sites

For definition of the terrestrial reference system



A Core Site defines terrestrial reference points in the space and time domain and in the gravity field of Earth.

Data from complementary instruments are used in a synergetic way

GNSS, SLR, VLBI, DORIS

Cesium Standards and hydro-maser

Absolute (AG) and Superconducting(SG) gravity meters.