On the estimation of the regional geoid error in Canada

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1. Introduction

With the increased use of GPS-based positioning, the demand for directly converting ellipsoidal heights to a height referred to a regional vertical datum with sufficient accuracy is also increasing. In Canada, the use of a gravimetric geoid as the national height reference surface has been under study. The recent revolutionary development of space gravimetry (i.e. CHAMP and GRACE) offers the opportunity to pursue such a drastic change in the local vertical datum. Among others, a compulsory requirement of adopting a vertical datum based on a gravimetric geoid, rather than conventional leveling networks, is a reliable assessment of the systematic and stochastic errors. This is a challenging task as there is limited or poor information regarding the quality of the terrestrial gravity data, difficulty in quantifying the gravity reduction and interpolation error, as well as only approximate estimates of the error associated with the satellite model. The purpose of this paper is twofold, namely (i) to provide a detailed examination of all sources of the gravimetric geoid error in Canada (e.g., satellite gravity model, terrestrial gravity data, gravity reduction methods, Stokes integration error and the far-zone contribution error), and (ii) to assess an adequate method for the estimation of the gravimetric geoid error from these error sources. In particular the variance component estimation scheme AUE is implemented to calibrate the geoid error for the Canadian Gravimetric Geoid 2005 (CGG05) using existing GPS-leveling data.

2. Methods

2.1 Geoid determination

$$N=\frac{G\delta M}{R\gamma}-\frac{\delta W}{\gamma}+N^{SG}_{H,2\sim l}+\frac{R}{4\pi\gamma}\int_{\Omega_0'}S_{DB}(\psi)(\Delta g_H-\Delta g^{SG}_{H,2\sim l})d\Omega'+F_N+PITE+c_e$$

$$F_N=N^{CG}_{H,l+1\sim m_{CG}}-\frac{R}{4\pi\gamma}\int_{\Omega_0'}S_{DB}(\psi)\Delta g^{CG}_{H,l+1\sim m_{CG}}d\Omega' \quad S_{DB}(\psi)=\sum_{n=l+1}^{m_{TG}}\frac{2n+1}{n-1}P_n(\cos\psi)$$

2.2 Error estimation

The geoid error is primarily comprised of the satellite and combined model errors as well as terrestrial gravity data errors: $v_N=v_{SG}+v_{TG}+v_{CG}$

$$v_{SG}=\frac{R}{2\gamma}\sum_{n=2}^{l}\left(\frac{2}{n-1}+Q_n^{DB}\right)\varepsilon_n^{SG} \qquad v_{TG}=\frac{R}{4\pi\gamma}\sum S_{DB}(\psi)\varepsilon^{TG}\Delta\Omega' \qquad v_{CG}=\frac{R}{2\gamma}\sum_{n=l+1}^{m_{CG}}Q_n^{DB}\varepsilon_n^{CG}$$

The co-variance matrix of the geoid heights can be expressed as:

$$\Sigma_N=\Sigma_{SG}+\Sigma_{TG}+\Sigma_{CG}$$

2.3 Estimation of variance components (VCE)

The differences between the GPS-leveling and the gravimetric geoid heights consist of systematic and random errors in the ellipsoidal height h, the orthometric height H and the gravimetric geoid N in terms of the following equation:

$$\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{v}+\mathbf{w}=0 \qquad \mathbf{w}=\mathbf{h}-\mathbf{H}-\mathbf{N}$$

$$\mathbf{v}=\begin{bmatrix}\mathbf{v}_h^T & \mathbf{v}_H^T & \mathbf{v}_{SG}^T & \mathbf{v}_{TG}^T & \mathbf{v}_{CG}^T\end{bmatrix}^T \qquad \mathbf{B}=\begin{bmatrix}\mathbf{I} & -\mathbf{I} & -\mathbf{I} & -\mathbf{I} & -\mathbf{I}\end{bmatrix}$$

Given a priori co-variance matrices for h, H and N, their variance components may be estimated using Almost Unbiased Estimation method (Fotopoulos 2003). Upon successful VCE, the error variances of N can be estimated.

3. Data

- The GRACE-based GGM02C model is used to compute the low-degree (degrees 2 to 90) geoid components and the far-zone contribution of the Stokes integration (degrees 91 to 200). The full co-variance matrix of GGM02C is used to compute the low-degree and far-zone contribution geoid errors.
- The terrestrial (ground, airborne, shipboard and satellite-altimetry-derived) gravity data are used to compute the high-degree (beyond degree 90) geoid components. Their error standard deviations were used to compute the high-degree geoid errors.
- The GPS data observed after 1994 are used to compute the ellipsoidal heights h with standard deviations of 1 – 2 cm.
- The leveling data observed after 1981 are used to compute the orthometric heights H along with gravity data and terrain corrections.

4. GPS-leveling h-H vs. the CGG05 gravimetric geoid N

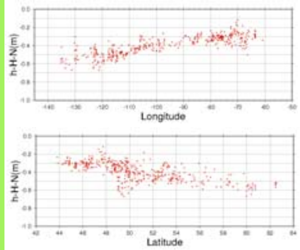


Figure 4.1 The h – H – N results.

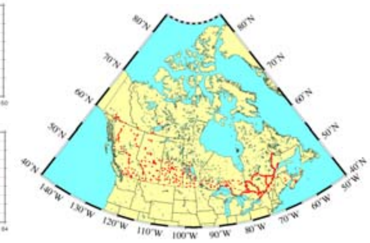


Figure 4.2 The GPS-leveling stations.

5. Co-variance matrices and VCE

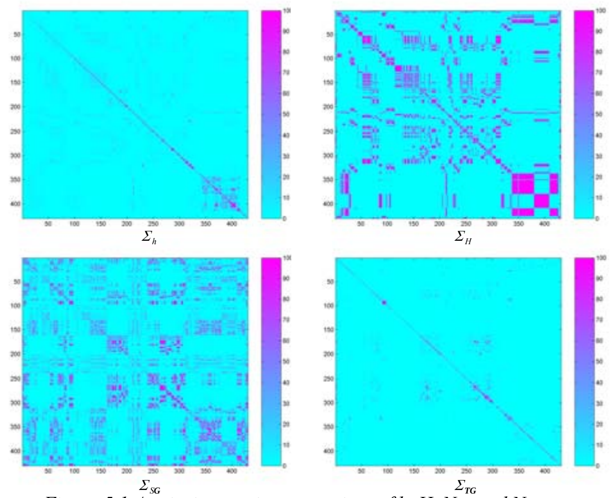


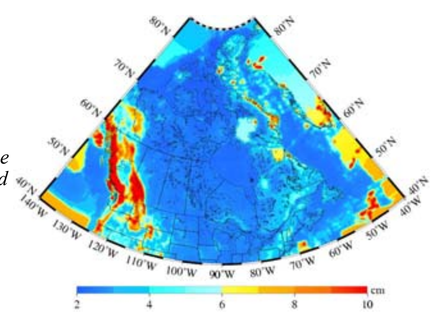
Figure 5.1 A priori co-variance matrices of h, H, N_{SG} and N_{TG} for the 430 GPS-leveling stations. Unit: mm².

The variance factor results:

Covariance Matrices	$\hat{\sigma}^2_h$	$\hat{\sigma}^2_H$	$\hat{\sigma}^2_{SG}$	$\hat{\sigma}^2_{CG}$	$\hat{\sigma}^2_{TG}$	Number of Iterations
Fully Populated	9.05	5.55	1.40	1.28	3.31	37
Diagonal Only	3.31	0.01	2.65	5.88	4.40	80

6. The total geoid error

Figure 6.1 The total gravimetric geoid error for CGG05 calibrated using the GPS–leveling data and their co-variance matrices over Canada and neighboring regions. Min = 2 cm, Max = 30.7 cm, Mean = 5.4 cm, and R.M.S. = 6.0 cm.



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